# No News is Good News: Voluntary Disclosure in the Face of Litigation[1]

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October 6, 2014

[1]We would like to thank Jeremy Bertomeu, Kyle Bagwell, Ron Dye, Ilan Guttman, Cam Harvey, Mike Harrison, Mirko Heinle, Jon Levin, Andy Skrypacz and workshop participants at Duke, CMU, EPFL, Wharton, Stanford, Universidad de Chile, and University of Houston for helpful comments.

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**Abstract**

This paper studies dynamic disclosure when the firm value evolves stochastically over time. The presence of litigation risk, arising from the failure to disclose unfavorable information, not only prompts bad news disclosures but also crowds out good news disclosures. The manager's disclosure policy and the overall amount of information transmission depend on the persistence of shocks, as managers may delay the release of negative information in an attempt to bet for resurrection. From a policy perspective, we show that a harsher legal environment may be a cost-effective way of stimulating information transmission in settings where the nature of information is highly proprietary.

*JEL Classification:* C73, D82, D83, D84.
*Keywords:* Voluntary Disclosure, Certification, Dynamic Games, Optimal Stopping.

# 1 Introduction

Firms receive information in an ongoing basis. Productivity shocks originate from various sources, including innovation breakthroughs, the arrival of new business opportunities, frictions in negotiations with labor unions, or break downs of supply-chain relationships. Capital market perceptions pressure managers to disclose their information frequently, because the stock price performance is affected by the firm's disclosures and lack thereof (see e.g., Graham, Harvey and Rajgopal (2005)).

In light of this fact, a firm's manager may feel inclined to disclose good news, but disclosing good news is often costly.[1] On the other hand, the manager may be tempted to delay the release of bad news, hoping that the firm prospects will improve at some point in the future. This *bet for resurrection* entails costs too: remaining silent for a long time could affect the evolution of the stock price, if investors grow wary that the manager might be concealing bad news. In addition, concealing bad news is risky because the information might be revealed by external sources, eventually triggering costly litigation. For example, In 2012 the U.S. Justice Department announced GlaxoSmithKline (GSK) had agreed to plead guilty and pay a $3 billion fine for withholding information about the cardiovascular risk of Avandia, GSK's antidiabetes drug.[2] Avandia's problems began in 2007 when a study published in the New England Journal of Medicine (Nissen and Wolski (2007)) found the drug carried a higher risk of heart attacks than alternative drugs. The 2012 settlement stems from claims made by four employees of GSK, who tipped off the U.S. government about GSK's concealment of two internal studies that preceded Nissen and Wolski (2007). Avandia prescriptions and GSK's stock price dropped sharply ever since the publication of Nissen and Wolski (2007).[3]

[1] It often exacerbates competition; it might require certification by a third party. Also, preparing and disseminating information often consumes management resources and entails legal fees.

[2] In addition to the settlement with the U.S. state and federal governments,GSK was forced to settle thousands of consumer lawsuits alleging that the drug had harmed their health, which in total cost Glaxo nearly $2.4 billion.

[3] In 2014 Toyota was forced to pay $1.2 billion penalty to settle the criminal probe into

This paper studies managers' incentive to disclose information when withholding negative information is costly due to litigation risk. How is the manager's disclosure policy affected by the possibility of litigation? When do managers delay the release of negative information? We provide a model that highlights a fundamental asymmetry between positive and negative information. When the market expects positive information to be disclosed, no news is bad news. The market punishes a firm that does not release any information. On the other hand, when the market expects negative information to be disclosed, no news is good news and the market rewards the absence of disclosure. This asymmetry is important because, unlike positive disclosures, which cannot be imitated, silence is something that firms can imitate. Hence, the presence of litigation risk allows firms to signal to the market good standing by delaying the release of information. We show that this trade off determines both the timing and content of disclosures.

We present a continuous time disclosure model. Specifically, we analyze a disclosure game between the manager of a firm and a mass of buyers (the market) when the firm's asset value evolves stochastically over time. The evolution of the asset value is described by a continuous-time Markov chain that fluctuates between two possible states: low asset value and high asset value. The manager can disclose his private information at any point in time and as many time as he so wishes. Unraveling is not possible in equilibrium because disclosing good news is costly (so the firm cannot do it continuously). Concealing bad news is risky because there is a public news process, with positive arrival rate when the asset value is low, that triggers costly litigation if the manager did not disclose the information prior to the news arrival. The market is competitive and sets the stock price of the firm continuously, based on all publicly available information. The manager maximizes the present value of the firm's future stock prices, perhaps because his compensation, at

its handling of unintended acceleration problems that led to recalls of 8.1 million vehicles in 2009. The attorney General Eric Holder called the settlement the largest U.S. criminal penalty ever imposed on a car company and asserted "we can say for certain that Toyota intentionally concealed information and misled the public about the safety issues behind these recalls."

each point in time, is proportional to the firm's stock price.

The presence of litigation risk suggests that managers will preempt bad news, by voluntarily disclosing the bad news, if any, to avoid litigation costs. This preemption strategy is well documented empirically (see e.g., Lev (1995), Skinner (1994); Johnson, Kasznik and Nelson (2001)), but not well understood in theory. In fact, the bad news preemption idea poses a conceptual difficulty: if markets expected the manager to reveal bad news, at a given point in time, then the manager's silence would be interpreted as a clear sign that the manager's information is favorable, which would in turn lead to an upward jump in the stock price. However, rewarding silence in this way cannot be part of an equilibrium because, unlike good news, which can be verified, remaining silent is something that all firms can do, including those in financial trouble. Our analysis reveals that in equilibrium the firm can release bad news but only probabilistically. Indeed, the equilibrium predicts that when falling stock prices reach a certain threshold, the firm will reveal bad news with a probability that depends on the arrival intensity of the public news, the cost of litigation, and the proprietary cost of disclosing good news. At that point, the stock price will remain constant for some time, until bad news are finally disclosed.

Litigation risk not only leads to preemption of bad news but, more importantly, it crowds-out disclosure of good news, because silence is interpreted, per se, as a favorable signal of the firm's prospects. The manager is able to reveal good news in two ways: he can either explicitly disclose good news and bear the cost, or signal the good news by remaining silent. There is a *pecking order*: the manager prefers to use silence when the firm's undervaluation is moderate, and to use disclosure when the firm's undervaluation is severe.

The presence of litigation risk may be desirable, even from the firm's perspective. By creating a new communication mechanism that allows managers to convey good news without disclosing them, the existence of litigation risk allows the firm to save on proprietary disclosure costs.[4] From a policy per-

[4] When adverse shocks are permanent the result is stronger: the manager may prefer a high litigation risk to zero litigation risk. Numerical computations suggest that this result

spective, this means that a harsher legal environment may be a cost-effective way of improving information transmission, especially in settings where the nature of information is highly proprietary.

Our model is stylized but tractable and allows us to study interesting applications.

In Section 5, we make **litigation risk endogenous** by considering the incentives to monitor the firm. For example, the False Claims Act in the U.S. encourages people with knowledge of suspected false claims to sue on the government's behalf. If the Justice Department joins one of these lawsuits, the plaintiff can receive 15% to 25% of recoveries. This law creates significant incentives for layman to monitor firms' disclosure behavior thereby leading to endogenous litigation risk.[5] To capture the endogeneity of litigation risk, we assume there is a whistleblower who may investigate the firm at a cost. The wistleblower receives a reward, paid by the firm, if he is able to establish that the firm concealed negative information. We demonstrate that the wistleblower tends to investigate firms with relative low prior performance for whom the market uncertainty is relatively large (the ones that have remained silent for a long period of time). The presence of the wistleblower means that, in equilibrium, there must always be a positive probability of concealment, no matter how large the litigation cost is. This complementarity between concealment and monitoring is reminiscent of the literature with two sided moral hazard (see e.g., Halac and Prat (2014))

In Section 6, we study the relation between **patents and disclosure rate**. We consider the case when the firm's cash flows have a finite, predetermined life, perhaps because the firm's asset is protected by a patent. In this setting, we show that firms tend to delay disclosures of adverse events until the patent's expiration is sufficiently close, and cluster the release of bad news just prior

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still holds if negative shocks are sufficiently persistent.

[5]An article published in the Wall Street Journal (7/24/2014) relates the story of Dr. William LaCorte, as a "serial whistleblower." Recently LaCorte received a $38 million cut under a federal law that encourages fraud reporting. Much was from a $250 million U.S. settlement with Merck in 2008 over allegations it overcharged Medicaid for Pepcid, a heartburn drug. He used part of his portion to buy a boat he named Pepsid.

to the patent's expiration. To the best of our knowledge these predictions are new and have not been tested yet. Also, we show that the longer the patent, the more slowly the firm reveals the adverse information. This intuitive result suggests that, from a policy perspective, the more socially relevant the disclosures of the firm, the shorter should be the length of the patent awarded to the firm.

Our results also apply to the problem of **product quality certification** (see e.g., Dranove and Jin (2010)). Indeed, the model can be interpreted as one in which a monopolist sells a product of unknown quality to a mass of buyers. At each point in time, buyers purchase the good of unobservable quality at a price that equals the good's expected quality. The monopolist has the option to certify the product's quality at a cost to influence the trajectory of future prices. This certification-like interpretation, originally adopted by Jovanovic (1982), highlights the parallel between corporate disclosures and quality certification.

The rest of the paper is organized as follows. Section 2 presents the setting. Section 3 analyzes the baseline model without litigation risk. Section 4 introduces litigation risk. Section 5 studies endogenous public news in settings where fact checkers monitor the firm's disclosure behavior. Finally, Section 6 analyses the case of an asset with finite maturity and study the the effect of patents on disclosures.

## 1.1 Related Literature

This paper extends Jovanovic (1982) and Verrecchia (1983, 1990) to a continuous time setting. Unlike existing literature, our model features a continuous flow of private information. Moreover, our model includes a continuous flow of public information and the presence of stochastic litigation costs.

The most closely related paper is Acharya, DeMarzo and Kremer (2011). They consider a dynamic version of Dye (1985), where the manager may be privately informed about the asset value. When informed, the manager may disclose his private information at one of two points in time: at the start of the game or at a known date right after a public news signal is released. If the manager's private information is not so favorable, waiting for news has positive option value, since the public signal might induce a higher price in the absence of disclosure than in its presence. By contrast, if the public signal turned out to be unfavorable, the manager could mitigate its negative price effect by disclosing his own private information. Their model explains clustering of disclosure in bad times: the less favorable the public signal, the higher the probability of disclosure.

Kremer, Guttman and Skrypacz (2012) consider disclosure timing and the resulting price consequences. They study a two-period extension of the Dye (1985) model, where in each period, the manager may observe with some probability any of two pieces of information (if previously unobserved). They show that later disclosures are interpreted more favorably by the market because, in equilibrium, when partial disclosures are made earlier, the probability that the manager is hiding information is perceived to be higher.

In the presence of litigation risk, beliefs dynamics resemble those in the recent literature on dynamic signaling. Like in previous dynamic signaling models (Bar-Isaac, 2003; Daley and Green, 2012; Gul and Pesendorfer, 2012; Lee and Liu, 2013), the use of mixed strategies introduces a lower belief threshold such that the decision to delay information prevents beliefs to fall even further.

This paper relates to the industrial organization literature on imperfect competition with uncertain quality (see e.g., Bagwell and Riordan (1991); Conlisk, Gerstner and Sobel (1984); Stokey (1981); Daughety and Reinganum (2008)). Although in this strand of the I.O literature, prices are chosen by firms as signals of product quality, in our paper the firm controls the price only indirectly by influencing buyers' beliefs through certification. For example, in Bagwell and Riordan (1991) high and declining prices signal high quality, instead in our paper this price pattern reflects a deterioration of market beliefs about quality.

The literature studying the effect of quality certification on consumer choices and seller behavior is also closely related. In a recent survey, Dranove and Jin (2010) argue that "while most existing studies have examined the short-run consequences of quality disclosure, little is known about long-run effects." This paper characterizes the distribution of steady state prices and the market's long-run uncertainty about quality.

This paper builds on the literature on the relationship of litigation risk and disclosure. Ongoing work by Dye (2014) extends Dye (1985) by incorporating litigation risk. Daughety and Reinganum (2008) study a static model where a firm facing litigation risk may either signal safety/quality via pricing or quality disclosures. The authors show that, depending on the level of litigation risk, the equilibrium may result in too little or too much disclosure.

Our endogenous litigation extension is closely related to the literature on monitoring and auditing. In particular, a series of early papers study the problem of monitoring when the monitor cannot commit to a monitoring strategy. Graetz, Reinganum and Wilde (1986) for example study the incentives of the IRS to audit taxpayers when they may underreport their taxable income and a fraction of the taxpayers are honest. Our endogenous monitoring extension, in Section 5, can be considered as a dynamic extension of Graetz, Reinganum and Wilde (1986). Our endogenous litigation extension is also closely related to Halac and Prat (2014). In their model, a principal can monitor a myopic agent's "good behavior" so as to mitigate the agent's tendency to shirk in the absence of "recognition". Whereas in our model the fact finder monitors the manager to extract rents from the manager's bad behaviour. Also, while in our model the fact finder chooses the intensity of monitoring on a continuous basis, in Halac and Prac (2014) monitoring is a lumpy investment that entails a fixed cost and can depreciate. Such an investment gives rise to a detection technology that produces a verifiable (Poisson) signal which has positive arrival intensity when the agent exerts effort. Hence, if the technology is in place, the intensity of monitoring is fixed.

# 2 Model

We consider a firm that pays a terminal dividend $V_{\tau_M}$ when the firm matures, at a random time $\tau_M$ that has arrival intensity $\gamma$. We assume that the firm generates no cash flows before maturing in order to abstract from the informational role of dividends and focus on disclosures.

The value of assets $V_t$ follows a continuous-time Markov chain with state space $\{0, 1\}$. The terminal dividend is thus equal to 1 if the value of assets is 1 at time $\tau_M$, and zero otherwise. The value of the asset jumps from 0 to 1 with intensity $\lambda_1$ while it jumps back from 1 to 0 with intensity $\lambda_0$. We can think of $\lambda_0$ as the frequency with which the asset suffers an impairment. When $\lambda_1 = 0$, this impairment is permanent, otherwise the impairement is transitory.

At the outset, the asset value is known to be 1, namely $V_0 = 1$.[6] From that point onwards, the manager privately observes any shock to the asset value. However, at any point in time, the manager can disclose his private information at a cost. This disclosure cost may arise from the proprietary nature of information (as in Verrecchia (1983)), the need to certify the information to make it credible (for example, hiring an auditor, as in Jovanovic (1982)), or simply from the opportunity cost of the time required to prepare and disseminate the information.[7] We assume that the disclosure cost varies with the value disclosed. In particular, the cost of disclosing information is $C > 0$ when the asset value is high and 0 when the asset value is low. Hence, disclosing bad news is costless.[8]

Managers' incentives to disclose private information at any point in time depend on the velocity with which the information will leak into the market via external sources (e.g., media coverage, analysts' recommendations, peer firms' disclosures) and the way the market interprets the absence of public information. Despite the enormous flow of public information that characterizes the U.S. market, some commentators argue that U.S. managers are particularly keen on providing "earnings' guidance." Public information is represented by a Poisson process $N = \{N_t\}_{t \geq 0}$. If the value of assets is low, $N$ has arrival

[6]Nothing changes if $V_0$ is private information at the start.

[7]A number of large investors such as Warren Buffett (1996) and analysts such as Candace Browning (2006), head of global research at Merrill Lynch, have called for managers to give up quarterly earnings guidance and hence avoid the myopic managerial behavior caused by attempts to meet market expectations.

[8]This assumption is not required for the results to hold; assuming that the cost of disclosure in the low state is $C_0 > 0$ would generate the same predictions.

rate $\mu$, whereas if the value of assets is high, then $N$ has arrival rate 0. Hence, observing an arrival is perfect evidence of low asset value, which we refer to as bad news.[9] [10]

We assume that manager is subject to legal liability. If bad news arrives and the manager has not yet disclosed that the asset value is low, then the manager bears a fine with positive probability. Let $\ell_t$ be a random variable that takes the value 1 one in the event the manager is found liable of withholding information, and zero otherwise. The manager's personal cost of legal liability is denoted $c_\ell$ while the probability of experiencing this cost is $q$ if the last time the manager disclosed information he disclosed good news and zero otherwise.[11] Hence, if the manager's latest disclosure was bad news, then the manager is safe from legal liability since he can claim he already disclosed the bad news.[12] We denote by $\theta := c_\ell q$ the expected legal cost of not disclosing negative information, conditional on a news arrival. [13] Prices are set in a Bayesian and risk-neutral manner. We normalize the market's interest rate to be zero. So if $d_t \in \{0, 1\}$ denotes the disclosure decision at time $t$ and $d = \{d_t\}_{t\geq 0}$ denotes the market's conjecture about the manager's disclosure strategy, then the firm's stock price, given the history of disclosures $\mathcal{F}_t$, is set

[9] When public information is noisy, managerial disclosures may be triggered by a news arrival, and be used by the manager as a means to counteract the sometimes adverse price effect of noisy news. This reactive-like disclosures generate clustering of disclosure in bad times (see Acharya, DeMarzo and Kremer (2011)). For simplicity, we abstract away from this effect and instead focus on the the case where a news arrival reveal the underlying state perfectly, without noise.

[10] We could also have considered the case with positive Poisson shocks, in which an arrival represents a positive cash flow generated by a breakthrough or innovation. The characterization of the equilibrium would be similar, and the main economic forces would remain the same. We focus on the bad news case to lay the groundwork for the case with legal liability in Section 4.

[11] Strictly speaking $c_\ell$ is the normalized legal cost. If $C_\ell$ is the cost then $c_\ell := (\gamma+\kappa)C_\ell/\gamma$.

[12] Large and sudden declines in stock price at the time of an information release increase the risk of litigation considerably, ex post (Alexander, 1991). This evidence is consistent with our representation of the information environment. All we need is that the litigation cost is smaller when the manager preempts the bad news relative to when the bad news are conveyed by the public news.

[13] Our setting can be also mapped into situations in which non-disclosure of negative information entails real costs such as those arising when the manager continues a project which has become unprofitable so to not reveal the market the bad news about the project.

as

$$P_t = E^d(V_{\tau_M}|\mathcal{F}_t) \tag{1}$$

where $E^d(\cdot)$ denotes the expectation operator based on the measure induced by $d$. Following Acharya, DeMarzo and Kremer (2011) and Benmelech, Kandel and Veronesi (2010), we assume the manager chooses a disclosure strategy $\sigma$ that maximizes the present value of future prices net of disclosure expenses and litigation costs:

$$\mathcal{U}_t(d,\sigma) := E^d\left[\int_t^{\tau_M} e^{-\rho(s-t)}P_s ds - C\sum_{t\leq s<\tau_M} e^{-\rho(s-t)}\sigma_s - C_\ell\int_t^{\infty} e^{-r(s-t)}\ell_s dN_s\middle|\mathcal{F}_t, V_t\right], \tag{2}$$

Hence, the manager cares not only about the current price implications of his disclosures but also the long-term ones. This concern for future prices is natural and supported by the vast evidence that managers' wealth is affected by their own firms' stock price. These incentives may arise as a means of inducing the manager to exert effort, in the spirit of Benmelech, Kandel and Veronesi (2010), but since our focus is on disclosure behavior we take the manager's incentives as given.

The manager also cares about the present value of disclosure expenses. As mentioned above, the literature has often interpreted $C$ as arising from the proprietary nature of the information.[14] For this interpretation to be literally valid in a dynamic context, $C$ should be priced; namely it should be incorporated into the price as part of the firm's future cash flows. This alternative formulation would be more complicated without adding much economic insight (the interested reader can find this alternative formulation in Appendix A). Hereafter, we adopt the above formulation and interpret $C$ as the proprietary

[14] Verrecchia (1983) for example argues that "the release of a variety of accounting statistics about a firm may be useful to competitors, shareholders, or employees in a way which is harmful to a firm's prospects. One example of this is the response of the United Auto Workers for fewer labor concessions in the face of an announcement by Chrysler Corporation's chairman that that firm's fortunes had improved. Other examples might include the reluctance of managers in certain highly competitive industries, such as personal computers or airlines, or certain politically sensitive industries, such as the oil industry or foreign automobile importers, to disclose favorable accounting data."

cost of positive disclosures.

We can compute the evolution of beliefs using Bayes' rule. In the absence of news arrivals beliefs evolve according to [15]

$$\dot{p}_t = f(p_t), \tag{3}$$

where

$$f(p) = \kappa(\bar{p} - p) + \mu p(1 - p), \tag{4}$$

and

$$\bar{p} := \frac{\lambda_1}{\lambda_0 + \lambda_1}$$

is the stationary probability that the value of the asset is 1 and $\kappa := \lambda_0 + \lambda_1$ represents the asset's mean reversion, namely the speed at which market belief reverts to the stationary point $\overline{p}$ absent disclosure. This measure is the reciprocal of the persistence of shocks. In the absence of disclosures and news, beliefs experience a downward drift toward the stationary level $\hat{p}$ as defined by $f(\hat{p}) = 0$, where

$$\hat{p} = \frac{1}{2}\left(1 - \frac{\kappa}{\mu}\right) + \sqrt{\frac{1}{4}\left(1 - \frac{\kappa}{\mu}\right)^2 + \frac{\kappa}{\mu}\bar{p}}. \tag{5}$$

The unconditional probability that $V_t = 1$ given an initial condition $p_0 = p$ is

$$\phi_t(p) := \bar{p} + e^{-\kappa t}\left(p - \bar{p}\right).$$

The price of the firm at time $t$ is given by

$$P_t = \int_t^\infty \phi_{s-t}(p_t)\gamma e^{-\gamma(s-t)} ds = \frac{\kappa}{\gamma + \kappa}\bar{p} + \frac{\gamma}{\gamma + \kappa}p_t. \tag{6}$$

The price $P_t$ is affine in beliefs $p_t$. Hence, from here onwards, we use the terms "price" and "beliefs" interchangeably. Moreover, defining $r := \rho + \gamma$ and the normalized cost $c := (\gamma + \kappa)C/\gamma$ and $c_\ell := (\gamma + \kappa)C_\ell/\gamma$, the manager's

[15] See Karlin and Taylor (1981, p. 144).

objective function can be re-written as

$$U_t(d,\sigma) := E\left[\int_t^\infty e^{-r(s-t)}p_s ds - c\sum_{s\geq t} e^{-r(s-t)}\sigma_s - c_\ell \int_t^\infty e^{-r(s-t)}\ell_s dN_s \middle| \mathcal{F}_t, V_t\right], \tag{7}$$

where $\mathcal{U}_t$ and $U_t$ satisfy the following relation

$$\mathcal{U}_t = \frac{\kappa}{r(\gamma+\kappa)}\bar{p} + \frac{\gamma}{\gamma+\kappa}U_t.$$

Hence, the manager's disclosure strategy $\sigma$ maximizes (7) given the asset value $V_t$ and the market's belief. In this model, an equilibrium is defined as follows.

**Definition 1.** *An equilibrium is a disclosure strategy $d = \{d_t\}_{t\geq 0}$ and a belief process $p = \{p_t\}_{t\geq 0}$ such that, for all $t \geq 0$,*

1. *The market belief is $p_t = E^d(V_t|\mathcal{F}_t)$*
2. *The disclosure strategy maximizes the manager's utility given the market beliefs, that is $d \in \arg\max_\sigma U_t(d,\sigma)$.*

Both conditions are standard. At every point in time, the price is set according to Bayes' rule, given the manager's strategy and history. Similarly, the manager's disclosure strategy maximizes the manager's expected utility at each point in time and for all possible histories. In this paper, we focus on Markov Perfect equilibria.

**Certification of Product Quality.** The model allows for many applications. One important application is the certification of product quality (for two excellent surveys see Milgrom (2008) and Dranove and Jin (2010)). Indeed, the model can be interpreted as one in which a monopolist sells a product of unknown quality to a mass of buyers. At each point in time, the buyers purchase the good of unobservable quality $v \in \{0,1\}$ at a price that equals the good's expected quality, $p_t$. The monopolist has the option to certify the product's quality at a cost $c$ in order to affect the trajectory of future prices.

If we normalize the good's production cost to be zero, then the present value of the firm's expected profits, given a certification strategy $\sigma$, is given by (7). This certification-like interpretation, originally adopted by Jovanovic (1982), highlights the parallel between corporate disclosures and quality certification.

# 3 Equilibrium without Litigation

As a benchmark, we consider the case without litigation risk ($c_\ell = 0$) first. Markov equilibria are characterized by a disclosure threshold $p_*$ such that[16]

$$d_t = \mathbf{1}_{\{p_{t-} \leq p_*\}} V_t.$$

That is, the manager discloses at time $t$ if and only if both the price is lower than or equal to $p_*$ and the value of asset is high.[17] Anticipating this strategy, the market expects no disclosure when the price is above $p_*$. As a consequence, for any $p_t > p_*$ the price evolves according to (3). By contrast, for $p_t \leq p_*$, we have $p_t = d_t$. That is, if the manager does not disclose his information when the price reaches the threshold $p_*$, then the market infers that the asset value is low. As a result, the price drops from $p_*$ to zero and remains there until the manager discloses good news again (i.e., $V_t = 1$). This happens as soon as the asset value returns to the high state.

The dynamics of market beliefs are noteworthy. At the beginning of the game, the price drifts down for some time until disclosing good news becomes profitable to the manager. At that point, the price jumps upward if the manager discloses good news or downwards if the manager withholds information. Kothari, Shu and Wysocki (2009) empirically document a similar pattern. They find evidence consistent with the view that managers tend to withhold

[16]We use the usual left limit notation $p_{t^-} := \lim_{s \uparrow t} p_s$.

[17]This characterization has empirical support. Indeed, the idea that the propensity of disclosure is negatively correlated with the level of stock prices is natural, and has been documented empirically. For example, Sletten (2012) argue that "stock price declines prompt managers to voluntarily disclose firm-value-related information (management forecasts) that was withheld prior to the decline because it was unfavorable but became favorable at a lower stock price."

bad news from investors and that prices tend to drift down, absent disclosure and jump upward upon the release of good news. Notice that the failure to disclose at $p_t = p_*$ is followed by a period where (i) the price remains flat for some (random) time and (ii) the information becomes symmetric. By contrast, the period following a disclosure is characterized by the price (mean) reverting towards its long-run value $\overline{p}$, and by the manager having private information about the true asset value.

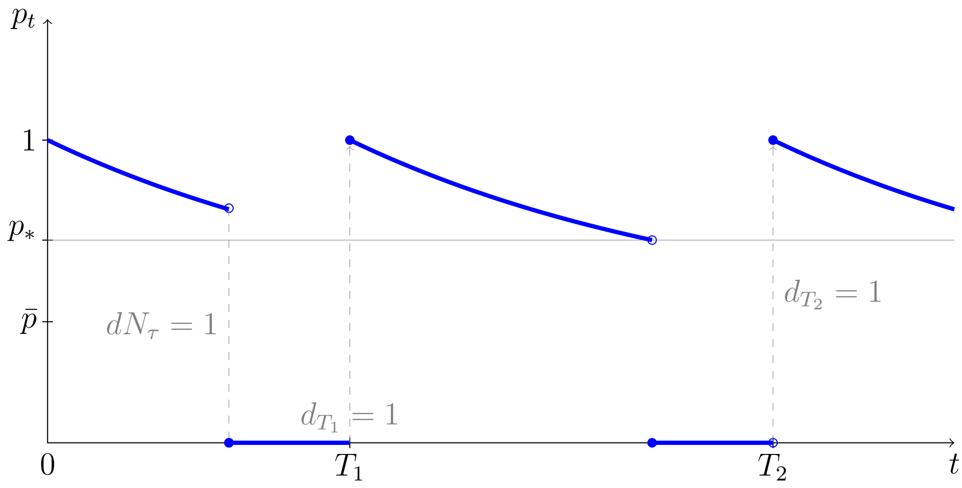



**Figure 1:** Example of a sample path of the share price.

In equilibrium, the market's conjecture $d$ must be consistent with the manager's optimal strategy $\sigma$. With some abuse of notation, let $U_v(p)$ be the manager's payoff given that the market belief is $p_t = p$ and the asset value is $V_t = v \in \{0, 1\}$. When $p > p_* > \hat{p}$, the manager's payoff in equilibrium can be represented by a an HJB equation

$$rU_v(p_t) = p_t + \frac{d}{dt}E\left[U_v(p_t)\right].$$

The interpretation of the value function is standard; we can think of the manager's job as an asset whose cost of capital in a competitive market $rU_v(p)$ must equal the rate of return on the asset, as given by its instantaneous flow $p$, and its expected capital gains $E\left[dU_v\right]/dt$. The latter may come in three forms: the deterministic evolution of investors' beliefs, the possibility the asset experiences a real negative shock, and the possibility that negative news arrives. The Hamilton-Jacobi-Bellman (HJB) equation is:

$$rU_1(p) = p + f(p)U_1'(p) + \lambda_0[U_0(p) - U_1(p)] \tag{8}$$

$$rU_0(p) = p + f(p)U_0'(p) + \lambda_1[U_1(p) - U_0(p)] + \mu[U_0(0) - U_0(p)] \tag{9}$$

with boundary conditions

$$U_1(p_*) = U_1(1) - c \tag{10}$$

$$U_0(p_*) = \frac{\lambda_1}{r + \lambda_1}[U_1(1) - c]. \tag{11}$$

Moreover, the value function must satisfy the following optimality conditions:

$$\begin{aligned} U_1(1) - c &\geq 0 \\ U_1(p) \geq U_1(1) - c &\quad \text{for} \quad p > p_* \\ U_1(p) \leq U_1(1) - c &\quad \text{for} \quad p \leq p_*. \end{aligned}$$

In essence, the manager must solve an optimal stopping problem where the stopping time is endogenous, since it must be consistent with the market's rational expectations.

In general, it is not possible to solve the HJB equation in closed form. The following proposition characterizes the equilibrium.

**Proposition 1.** *For any $p_* \in (\hat{p}, 1)$ satisfying*

$$U_1(1) - c \geq 0 \tag{12}$$

*and*

$$U_1'(p_*) \geq 0 \tag{13}$$

*there is an equilibrium with threshold $p_*$.*

It is instructive to consider the manager's payoff at the start of the game, namely when the market beliefs are $p = 1$. Let's define

$$\mathcal{C}(c) := \frac{\delta(1)}{1-\delta(1)}c.$$

Hence, the manager's payoff at the outset is given by

$$U_1(1) = U^{ND}(1) - \mathcal{C}(c).$$

The first component, $U^{ND}(1)$, is the payoff the manager would obtain had he been able to commit to never disclose.[18] The second component $\mathcal{C}(c)$ is the present value of the disclosure expense the manager expects to bear over his whole lifetime, given his lack of commitment.[19] The manager's payoff is thus bounded above by the non-disclosure payoff $U^{ND}(1)$. This is natural: in our setting, information has no (social) value, hence the disclosure expense is a deadweight loss borne by the manager only because he cannot avoid disclosing his information when the price is severely depressed. But ex-ante, the average trajectory of future prices is unaffected by the manager's disclosure policy; in equilibrium the event of disclosure drives the price up and the lack thereof drives the price down.

The discrete support of $V_t$ results in the existence of multiple equilibria. In particular, when the cost of disclosure is moderate, there exists a continuum of thresholds $p_*$ satisfying the equilibrium conditions. However, in our model, equilibria can be Pareto ordered; Harsanyi (1964) and Fudenberg and Tirole (1985) argue that confronted with two possible equilibria it is natural to focus on the Pareto-dominant one.

**Definition 2.** *The equilibrium threshold $p_*^\dagger$ is Pareto dominant if and only if $U_v(p|p_*^\dagger) \geq U_v(p|p_*)$ for all $p \in [0,1], p_* \in [p_*^-, p_*^+]$ and $v \in \{0,1\}$.*

[18] Weak commitments are sometimes observed in the real world. On December 13, 2002, the Coca Cola Company announced that it would stop providing quarterly earnings-per-share guidance to stock analysts, stating that the company hoped the move would focus investor attention on long-run performance.

[19] As a mirror image, one can think of this term as the profits of a certifier who, at the outset, commits to selling his certification services for a fee $c$.

This selection criterion is very weak since it requires the equilibrium to be Pareto optimal for all beliefs and all states. We can think of the Pareto-dominating equilibrium as the natural outcome of an extended game when, at the outset, the manager informally announces the firm's disclosure policy to the market. Though the manager cannot fully commit to disclosing regularly, he can issue a cheap talk message along the lines of "we will try to provide guidance on a quarterly basis".[20] This type of announcement is common in practice; firms often announce their disclosure policy ex ante. Although these announcements are non binding, they still help set market's expectations about firm's disclosure policies.

We have the following proposition describing the set of equilibria and the Pareto dominating one.

**Proposition 2.** *Suppose there are equilibrium disclosure thresholds $\hat{p} \leq p_*^- < p_*^+$ such that*

$$U_1\left(1|p_*^+\right) - c = 0 \tag{14}$$

$$U_1'\left(p_*^-|p_*^-\right) = 0, \tag{15}$$

*then, $p_*$ is an equilibrium disclosure threshold if and only if $p_* \in [p_*^-, p_*^+]$. Moreover, the least transparent equilibrium, $p_*^-$, is the Pareto dominant equilibrium.*

This result is intuitive. Given that disclosure is a deadweight cost, the most efficient equilibrium is the one that minimizes the frequency of disclosure, for this equilibrium minimizes also the present value of future disclosure expenses. The *most transparent* equilibrium, in terms of the probability of disclosure, arises when condition (12) is binding. By contrast, the *most opaque* equilibrium arises when condition (13) is binding.

[20] For example, Chen, Matsumoto and Rajgopal (2011) note that on December 13, 2002, the Coca Cola Company announced that it would stop providing quarterly earnings-per-share guidance to stock analysts, stating that the company hopes the move would focus investor attention on long-run performance. Shortly thereafter, several other prominent firms such as AT&T and McDonalds made similar announcements renouncing quarterly earnings guidance.

Notice again that the least transparent equilibrium is the manager's preferred equilibrium for any initial belief $p$ and any asset value. Hence the manager's incentives to coordinate in the least transparent equilibrium will remain the same for all histories of the game.

## 3.1 No Public News: $\mu = 0$

In general, it is not possible to solve the HJB equations in closed form. However, a closed form characterization exists for the case when $\mu = 0$. We refer to an equilibrium in which disclosure happens with probability zero (at any point in time and for any history) as a non-disclosure equilibrium. With some abuse of notation, we let $U_v(p|p_*)$ be the manager's expected payoff in an equilibrium with disclosure threshold $p_*$, when the state is $v$ and the current price is $p$. The solution to the HJB equation for a given disclosure threshold $p_*$ is

**Proposition 3.** *Suppose that $p_* \in (\bar{p}, 1)$, then the manager's payoff is given by*

$$U_0(p) = U_1(p) - \frac{r}{r+\lambda_1}\left(\frac{p_* - \bar{p}}{p - \bar{p}}\right)^{1+\frac{r}{\kappa}} \Big(U_1(1) - c\Big) \tag{16}$$

$$U_1(p) = \int_0^{T(p)} e^{-rt}\phi_t(p)dt + \delta(p)\Big(U_1(1) - c\Big), \tag{17}$$

*where*

$$U_1(1) = U^{ND}(1) - \frac{\delta(1)}{1-\delta(1)}c$$

*and*

$$\delta(p) := \left(\frac{p_* - \bar{p}}{p - \bar{p}}\right)^{\frac{r}{\kappa}} \left[\frac{r\bar{p} + \kappa\bar{p}}{r + \kappa\bar{p}} + \frac{r(1-\bar{p})}{r+\kappa\bar{p}}\frac{p_* - \bar{p}}{p - \bar{p}}\right],$$

$$T(p) = -\frac{1}{\kappa}\log\left(\frac{p_* - \bar{p}}{p - \bar{p}}\right).$$

The following proposition characterizes the equilibrium set

**Proposition 4.** *Let*

$$\hat{c} := \frac{\lambda_1 + r}{r\,(r+\kappa)}.$$

*If $c < (1-\bar{p})\hat{c}$, then any equilibrium has a positive probability of disclosure. In particular:*

1. *If $c < (1-\bar{p})\hat{c}$, there are disclosure thresholds $p_*^- < p_*^+$ satisfying the boundary conditions*

$$U_1\left(1|p_*^+\right) - c = 0 \tag{18}$$

$$U_1'\left(p_*^-|p_*^-\right) = 0, \tag{19}$$

   *such that, for any $p_* \in [p_*^-, p_*^+]$, there is an equilibrium with disclosure threshold $p_*$.*

2. *If $(1-\bar{p})\hat{c} \leq c < \hat{c}$, then for any $p_* \in [\bar{p}, p_*^+]$, where $p_*^+$ satisfies (18), there is an equilibrium with disclosure threshold $p_*$.*

3. *If $c \geq \hat{c}$, the only equilibrium entails no disclosure.*

*If $c < (1-\bar{p})\hat{c}$, then the Pareto dominant equilibrium is the least transparent equilibrium, that is, $p_*^\dagger = p_*^-$. On the other hand, if $c \geq (1-\bar{p})\hat{c}$, then the Pareto dominant equilibrium has no disclosure.*

This result reveals that the persistence of cash flows, $\kappa$, is a key determinant of disclosure frequency. For a given disclosure policy a higher persistence mitigates the price drift thus decreasing the frequency of disclosure. But persistence also affects the manager's disclosure incentives. A higher persistence means the the information is more long-lived and, therefore, any disclosure has a more long lasting effect on the stock price. The presence of these countervailing effects explains why the effect of persistence on the frequency of disclosure is non-monotonic.

# 4 Legal Liability and Disclosure of Bad News

In the real world, concealing bad news can be very costly. For example, in a recent case, Toyota was forced to pay $1.2 billion penalty to settle the criminal probe into its handling of unintended acceleration problems that led to recalls of 8.1 million vehicles beginning in 2009.[21] Similarly, in 2012, GlaxoSmithKline agreed to plead guilty and pay a $3 billion fine for withholding results regarding the cardiovascular safety of Avandia.[22]

The empirical literature argues that litigation risk is indeed an important driver of firms' voluntary disclosures. For example, Lev (1995) and Skinner (1994) document that managers can reduce stockholder litigation costs by voluntarily disclosing adverse earnings news "early," namely before the mandated release date. Consistent with this view, Skinner (1994) finds that managers use voluntary disclosures to preempt large negative earnings surprises more often than other types of earnings news.[23] In this section, we study the effect of litigation risk on disclosure patterns. We first consider the special case of permanent shocks, which is particularly tractable, and then the general case of transitory shocks.

## 4.1 Permanent Shocks: When Preemption Pays Off

Assume that if the asset experiences a negative shock, its value remain at zero forever; in other words, impairments are permanent (i.e., $\lambda_1 = 0$). Most of the insights on the effect of litigation risk can be captured in this stylized setting. Also, the assumption of permanent impairments is realistic in many real world situations. For example, permanent impairments arise when a regulator bans a pharmaceutical company from commercializing a drug because of safety con-

[21] The attorney General Eric Holder called the settlement the largest U.S. criminal penalty ever imposed on a car company and asserted "we can say for certain that Toyota intentionally concealed information and misled the public about the safety issues behind these recalls."

[22] The case of GSK's diabetes drug Avandia is paradigmatic. Its sales were $2.5-billion in 2006; however, following a study published in the New England Journal of Medicine in 2007 that linked the drug's use to an increased risk of heart attack, sales plummeted to $9.5-million in 2012. In 2012, the U.S. Justice Department announced GSK had agreed to plead guilty and pay a $3 billion fine, in part for withholding the results of two studies of the cardiovascular safety of Avandia between 2001 and 2007 (New York Times, July 2, 2012).

[23] Also, Skinner (1997) finds that voluntary disclosures occur more frequently during quarters that result in litigation than in quarters that do not, because managers' incentives to pre-disclose earnings news increase as the news becomes more adverse, presumably because this reduces the cost of resolving litigation that inevitably follows in bad news quarters.

cerns; a borrower defaults on its debt, or technological improvements drive a product out of the market.

Some preliminary analysis and notational conventions are in order. The evolution of prices, absent public news, is given by

$$\phi_t(p_0) = \frac{p_0(\lambda_0 - \mu)}{e^{(\lambda_0-\mu)t}(\lambda_0 - \mu(1-p_0)) - p_0\mu}.$$

when the initial price is $p_0$. Using this equation we can define $T(p_0; p_*)$ as the solution of $\phi_T(p_0) = p_*$. The function $T(p_0; p_*)$ thus represents the time required for the price $p_t$ to reach $p_*$ when the initial price is $p_0$. This number can be computed as

$$T(p_0; p_*) = \frac{1}{\lambda_0 - \mu} \ln\left(\frac{p_0}{p_*}\frac{\lambda_0 - \mu + p_*\mu}{\lambda_0 - \mu + p_0\mu}\right)$$

As in previous sections, absent news and disclosures, the price drifts downward due to the possibility of an undisclosed impairment, but the drift is mitigated by the intensity of public news $\mu$. Though the asset's long term value is $\overline{p} = 0$, the price absent news and disclosure is bounded from below by

$$\hat{p} = \max\left(1 - \frac{\lambda_0}{\mu}, 0\right)$$

In the sequel, we restrict attention to parameter values such that $\hat{p} < \mu\theta$, for otherwise disclosure of bad news would never occur in equilibrium. Similarly, we assume $\mu\theta < 1$ for otherwise bad news would be fully disclosed and the information would be symmetric at each point in time.

Depending on the cost of disclosure $c$, different equilibrium structures emerge. However, all equilibria are characterized by a threshold $p_*$ such that whenever the price reaches the threshold, the manager may disclose some of his information. Whether the manager discloses good or bad news when $p_t = p_*$ depends on the magnitude of the disclosure cost relative to the litigation cost $\mu\theta$. In the sequel, we refer to a *good news equilibrium* as that arising when the manager discloses good news at $p_t = p_*$, and a *bad news equilibrium* as that arising when the manager (may) only disclose bad news at $p_*$. We first characterize the bad news equilibrium, and then we provide the whole taxonomy of equilibria.

At first blush, one might think that the presence of litigation risk ($\theta > 0$) would lead the manager to spontaneously disclose bad news, especially when prices are relatively low. But on closer inspection, this is not so clear: if the market expected the manager to disclose bad news at a particular point in time, then withholding the information at that point would be interpreted by the market as clear evidence that the asset value is high. This would lead to a jump in the stock price, which in turn would destroy the manager's incentives to disclose the information in the first place. The temptation to withhold bad news, so to benefit from the price jump, would offset the litigation preemption benefits from disclosing such news. This suggests that the manager's disclosure strategy must entail randomization.

Consider the bad news equilibrium. Assume the manager discloses bad news when the price reaches a threshold $p_*$ but never discloses good news (given that $\lambda_1 = 0$). We guess and verify that any equilibrium where bad news are disclosed with a positive probability has the following structure. If $V_t = 0$, then:

1. If $p_t > p_*$ we have $d_t = \emptyset$.

2. If $p_t = p_*$ then the manager discloses at an exponential time with a mean arrival rate

$$\zeta = \kappa \frac{p_*}{p_*(1 - p_*)} - \mu.$$

The probability of disclosure $\zeta$ is such that the price process, $p_t$, has a lower barrier at $p_*$. Figure 2 shows a sample path of the stock price in equilibrium. At the outset, the price experiences a downward drift up until it reaches $p_*$. If the asset value is low when the price reaches $p_*$, then the manager randomizes between disclosing and not disclosing his information. In response, the price remains flat for some time, until the manager reports bad news, at time $T_1$. Naturally, such a disclosure causes the price to drop and stay at zero forever, given that the impairment is permanent.

Unlike the case without litigation risk analyzed in Sections 2, here the manager strictly prefers not to disclose good news. This speaks to the notion that litigation risk crowds out disclosure of good news: the presence of litigation risk not only prompts the manager to disclose bad news but also removes the incentive to disclose good news. The reason is that the absence of good news disclosures is now perceived by the market as a favorable signal of asset value: because the market expects that the manager will sometimes disclose bad news, it also interprets more favorably the act of withholding information. This effect is sometimes strong enough to fully offset the drift in the stock price.

As previously mentioned, the equilibrium includes randomization. The manager's randomization keeps the price from jumping upward when the price reaches the threshold, but forces it to either remain constant in the absence of disclosure, or to drop to zero in the presence of a disclosure (see Figure 2). Of course, for randomization to be the manager's optimal response, he must be indifferent between disclosing low values to avoid the risk of litigation, and not concealing the bad news to enjoy inflated prices.

The threshold $p_*$ characterizes an optimal disclosure strategy if the manager's payoff satisfies the following HJB equation. For $p_t > p_*$,

$$rU_1(p) = p + f(p)U_1'(p) + \lambda_0[U_0(p) - U_1(p)] \tag{20}$$

$$rU_0(p) = p - \mu\theta + f(p)U_0'(p) + \mu[U_0(0) - U_0(p)]. \tag{21}$$

These equations are analogous to those encountered in previous settings, except that in the low state, the manager's instant payoff is given by the price net of expected litigation costs. These equations also show that the manager is exposed to two types of shocks. First, the asset value may experience a "real shock" that even when not observed by the market, affects both the trajectory of prices and expected litigation costs. Second, the manager may experience a "news shock"; a news arrival may reveal that the manager withheld information, thus triggering a drop in the stock price and potential litigation costs.

The boundary conditions are straightforward in this case. First observe that

$$U_0(0) = 0,$$

given that the shocks are permanent. Also, for the manager to be willing to randomize at $p_*$ we have

$$U_0(p_*) = U_0(0).$$

This condition allows us to pin down uniquely the disclosure threshold. Hence, unlike good news equilibria, no selection criterion is required.

**Lemma 1.** *The solution to the HJB equation, when the initial price is $p$ and the equilibrium threshold is $p_*$, is given by*

$$\begin{aligned} U_0(p) &= \int_0^{T(p)} e^{-(r+\mu)t} \big(\phi_t(p) - \mu\theta\big) dt \\ U_1(p) &= \int_0^{T(p)} e^{-(r+\lambda_0)t} \left( \frac{\mu - \lambda_0 e^{-(\mu-\lambda_0)t}}{\mu - \lambda_0} \phi_t(p) - \frac{1 - e^{-(r+\mu)(T(p)-t)}}{r+\mu} \lambda_0 \mu\theta \right) dt \\ &\quad + e^{-(r+\lambda_0)T(p)} U_1(p_*). \end{aligned}$$

*where $T(p) = T(p, p_*)$.*

Inspection of these equations reveal that the threshold $p_*$ is consistent with the manager's optimal disclosure policy if and only if

$$p_* = \mu\theta.$$

The manager's disclosure policy is thus myopic. This is an artifact of $\lambda_1 = 0$. In the general case, i.e., when $\lambda_1 > 0$, the equilibrium threshold is always lower than $\mu\theta$ because delaying disclosures, as an attempt to *bet for resurrection*, has some option value.

We can now determine the propensity with which the manager discloses bad news. Using Bayes' rule, we find that the value of $\zeta$ that ensures the price remains flat at $p_t = \mu\theta$, absent disclosure, is given by

$$\zeta = \frac{\lambda_0 \mu\theta}{\mu\theta(1-\mu\theta)} - \mu.$$

The propensity of disclosure $\zeta$ increases monotonically in the cost of litigation $\theta$ and the likelihood of a shock $\lambda_0$, but it is u-shaped in the intensity of public news $\mu$.

So far, we have only considered the bad news equilibrium. Depending on the importance of litigation risks vis-a-vis disclosure costs, there may be no bad news equilibrium or the bad news equilibrium may coexist with good news equilibria. In these cases, we resort to Pareto optimality (see Definition 2) as the refinement criterion.

The next proposition provides the taxonomy of equilibria.

**Proposition 5.** *Let $\underline{c} < \bar{c}$ be defined as*

$$\underline{c} := \left[1 - \left(\frac{\mu\theta\lambda_0}{\lambda_0 - \mu + \mu^2\theta}\right)^{\frac{r+\lambda_0}{\lambda_0-\mu}}\right] \frac{r + \mu - (r + \mu + \lambda_0)\mu\theta}{(r+\mu)\,(r+\lambda_0)} + \left(\frac{\mu\theta\lambda_0}{\lambda_0 - \mu + \mu^2\theta}\right)^{\frac{r+\lambda_0}{\lambda_0-\mu}} \frac{1-\mu\theta}{r+\mu}$$

$$\bar{c} := \frac{(r + \mu - \lambda_0\mu\theta) - (\mu\theta r + \mu - \lambda_0)\left(\frac{\mu\theta\lambda_0}{\lambda_0-\mu+\mu^2\theta}\right)^{\frac{r+\lambda_0}{\lambda_0-\mu}}}{(r+\lambda_0)\,(r+\mu)}.$$

*Then,*

1. *An equilibrium with disclosure of good news exists if and only if $c \leq \bar{c}$.*

2. *An equilibrium with disclosure of bad news exists if and only if $c \geq \underline{c}$.*

3. *If $c < \underline{c}$, then any equilibria with good news has a threshold strictly greater than $\mu\theta$. That is, $p_*^- > \mu\theta$.*

4. *If $c < \underline{c}$, the Pareto-dominating equilibrium is the equilibrium with disclosure of good news and threshold $p_*^- > \mu\theta$. Alternatively, if $c \geq \underline{c}$, the Pareto-dominating equilibrium is the equilibrium with disclosure of bad news with threshold $\mu\theta$.*

*Proof.* The proof can be found in the appendix in a sequence of Lemmas. Part 1 is proven in Lemma 13. Part 2 is proven in Lemma 15. Finally, parts 3 and 4 are proven in Lemma 17. ∎

This taxonomy is intuitive. For very low disclosure costs, only good news can be released in equilibrium because, otherwise, the manager would have an incentive to disclose good news before the price reaches the threshold $\mu\theta$. Conversely, for very high disclosure cost, only the bad news equilibrium can prevail. Otherwise, in a good news equilibrium, the manager would have an incentive to preempt bad news before the price reaches the equilibrium threshold. For intermediate values of $c$, both type of equilibria may coexist, but the bad news equilibrium Pareto-dominates even the most efficient good news equilibrium. Remarkably, upon imposing Pareto dominance, we end up with a unique equilibrium for the entire range of parameters.

The above taxonomy could be alternatively described based on the litigation risk ($\mu$ or $\theta$) rather than the disclosure cost $c$. This alternative taxonomy would show that, as litigation risk goes up, the equilibrium shifts from good news to bad news disclosures. This substitution in the type of disclosure, and in particular the lower probability of good news disclosure caused by the higher litigation risk, has some empirical support (Lev (1995); Johnson, Kasznik and Nelson (2001)), and has been explained as arising from the fact that litigation is triggered by "optimistic disclosures" (Lev (1995)). We provide an alternative (perhaps complementary) explanation: litigation risk crowds out good news because silence is perceived by itself as a favorable signal when withholding bad news entails such risk.

**Discussion** The empirical literature identifies litigation risk as an important determinant of corporate disclosures (see e.g., Skinner (1994); Johnson, Kasznik and Nelson (2001)), but the sign of the effect is not clear (for example, Francis, Philbrick and Schipper (1994) argue that litigation potentially reduces incentives to provide disclosure, particularly of forward-looking information). However, because litigation risk is not directly observable, this literature has resorted to measuring litigation risk indirectly from the ex-post frequency of litigation; namely the number of class action lawsuits observed in a given industry or among firms of a certain class. In our setting, this is potentially misleading –the frequency of disclosure and the likelihood of litigation are simultaneously determined; they are not causally linked. In practice, we might observe a lower frequency of litigation among firms facing greater litigation risks, simply because these firms preempt litigation more often, by disclosing their bad information quickly. Conversely, we might observe a higher frequency of disclosure among firms facing more litigation simply because they are exposed to tighter scrutiny from the market.

Another relevant question is whether litigation risk is "undesirable" from the manager's perspective, as intuition would suggest. On the contrary, we find that a higher litigation risk may improve the manager's welfare. The reason is intuitive: as mentioned above, higher litigation risk may crowd out good news disclosures, thereby reducing the firm's overall disclosure expense. The manager may benefit from a higher litigation risk simply because he lacks commitment power to avoid incurring proprietary disclosure costs, and the presence of litigation risk may remove this incentive altogether. Notice that the manager's ex-ante welfare is greater when $\theta$ is very large vs. when $\theta$ approaches zero. While in the former case the manager fully preempts bad news at no cost, in the latter case the manager incurs proprietary disclosure costs by disclosing good news. We can show that for high litigation risk, the manager is better off in the presence of litigation risk than in its absence.

**Corollary 1.** *If $c < (1-\bar{p})\bar{c}$, then there is $\underline{\theta} < 1/\mu$ such that for all $\theta \geq \underline{\theta}$, the manager's expected payoff at time 0, $U_1(1)$, is strictly higher in the presence of litigation.*

*Proof.* From Proposition 3 we know that in absence of litigation risk the manager ex-ante payoff is $U_1^{nl}(1) = U^{ND}(1) - c\delta(1)/(1-\delta(1))$. If we take $\theta \geq 1/\mu$ we get $p_* = 1$ so the manager's payoff with litigation risk is $U^l(1) = 1/(r+\lambda_0) = U^{ND}(1)$. Accordingly, in this case, the manager payoff is strictly higher in the presence of litigation risk. By continuity of $U^l(1)$ with respect to $\theta$ there is $\underline{\theta} < 1/\mu$, such that the inequality continue to hold. ∎

## 4.2 Temporary Shocks: Betting for Resurrection

The case of permanent shocks is tractable but somewhat restrictive, for the following two reasons. First, good and bad news cannot coexist in equilibrium. Second, in the bad news equilibrium, the manager adopts a myopic policy which ignores the option value associated with delaying bad news. By contrast, the case of temporary shocks features both aspects: first, the manager may disclose good and bad news, second the manager's disclosure policy is not myopic; the manager is willing to delay disclosures of bad news for some time, and even bear temporary losses, hoping that a positive shock would render the disclosure of bad news unnecessary.

In this section, we restrict attention to the bad news equilibrium. We again conjecture and verify that the equilibrium is given by

1. If $V_t = 1$, then $d_t = \mathbf{1}_{\{p_{t^-}=0\}}$.
2. If $V_t = 0$, then:
   (a) If $p_{t^-} > p_*$ we have $d_t = \emptyset$.
   (b) If $p_{t^-} = p_*$ then the manager discloses with a mean arrival rate
   $$\zeta = \kappa \frac{p_* - \bar{p}}{p_*(1-p_*)} - \mu.$$
   (c) If $p_{t^-} < p_*$ then the manager discloses immediately with probability[24]
   $$\frac{p_{t^-}}{1-p_{t^-}} \frac{1-p_*}{p_*}.$$

Figure 2 shows a sample path of the stock price in equilibrium. At the start, the price experiences a downward drift until it hits the threshold $p_*$. This downward drift is caused purely by the increased likelihood of an undisclosed impairment. Given bad news, the manager starts randomizing between

[24]This is an out-off-equilibrium event. With perfect bad news, the market beliefs never enter the interval $(0, p_*)$ on the equilibrium path.

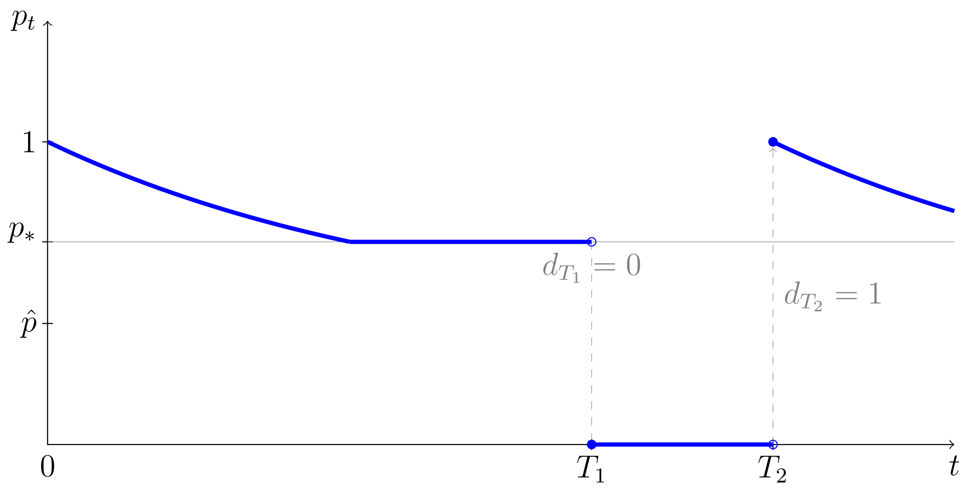



**Figure 2:** Example of a sample path of the share price with litigation cost.

disclosing and not disclosing his private information. The price remains flat until the manager reports bad news at time $T_1$. Naturally, this disclosure causes the price to drop to zero and stay there until the situation of the firm improves, at time $T_2$, and the manager discloses good news. The empirical evidence is broadly consistent with this price pattern. For example, Kothari, Shu and Wysocki (2009) find that bad news (good news) disclosures are preceded by a downward (upward) drift in the stock price.

The threshold $p_*$ characterizes an optimal disclosure strategy if the manager's payoff satisfy the following HJB equation. For $p_t > p_*$,

$$
\begin{aligned}
rU_1(p) &= p + f(p)U_1'(p) + \lambda_0[U_0(p) - U_1(p)] \\
rU_0(p) &= p - \mu\theta + f(p)U_0'(p) + \lambda_1[U_1(p) - U_0(p)] + \mu[U_0(0) - U_0(p)].
\end{aligned}
$$

The manager's decision to disclose bad news has the flavor of the real options problem analyzed by Dixit (1989), where a firm has the option, at any point in time, to shut down a project (i.e., disclose bad news) or restart it (i.e., disclose good news), based on the project's observed profitability. The difference is that the payoffs are endogenous in our analysis, because they are linked to the market's equilibrium belief about asset values. When the stock price is low (and asset value is low), disclosing bad news becomes profitable for the same reason shutting a loss making project is optimal in Dixit's model. Also, as in Dixit's problem, the decision to disclose bad news today is linked to the option to disclose future good news: if the likelihood of disclosing good news in the future goes down (perhaps because $\lambda_1$ is smaller, or the proprietary costs are higher), then the manager's incentive to disclose bad news today weakens. Consequently, he further delays such disclosures. This speaks to a certain complementarity between disclosure of bad news and good news.

To complete the characterization of the equilibrium, we need to derive the boundary conditions. When $p_t = p_*$, we have

$$U_0(p_*) = E\left[\int_0^{\tau_N\wedge\tau_D\wedge\tau_1} e^{-rt}(p_* - \mu\theta)dt + e^{-r\tau_N\wedge\tau_D\wedge\tau_1}\big(U_0(0)\mathbf{1}_{\{\tau_N\wedge\tau_D<\tau_1\}} + U_1(p_*)\mathbf{1}_{\{\tau_N\wedge\tau_D>\tau_1\}}\big)\right],$$

where $\tau_N$ is the first arrival of public (bad) news, $\tau_D$ is the time at which the manager voluntarily discloses bad news, and $\tau_1$ is the time at which the value of assets jump from 0 to 1. We can solve for the expected payoff of a low type manager, as given by

$$\begin{aligned} U_0(p_*) &= \int_0^{\infty} e^{-(r+\mu+\zeta+\lambda_1)t}\big(p_* - \mu\theta + (\mu+\zeta)U_0(0) + \lambda_1 U_1(p_*)\big)dt \\ U_0(p_*) &= \frac{p_* - \mu\theta}{r+\mu+\lambda_1+\zeta} + \frac{\mu+\zeta}{r+\mu+\lambda_1+\zeta}U_0(0) + \frac{\lambda_1}{r+\mu+\lambda_1+\zeta}U_1(p_*). \end{aligned} \tag{22}$$

Following similar steps as the ones above, we get the boundary condition for a high type manager as given by

$$U_1(p_*) = \frac{p_* + \lambda_0 U_0(p_*)}{r+\lambda_0}. \tag{23}$$

In addition, we have the following conditions when $p_t = 0$:

$$U_0(0) = \frac{\lambda_1}{r+\lambda_1}U_1(0) \tag{24}$$

$$U_1(0) = U_1(1) - c. \tag{25}$$

As the manager is using a mixed strategy when $p_t = p_*$, he must be indifferent between disclosing and not disclosing negative information, otherwise he would not be willing to randomize. Hence, we can determine the threshold $p_*$ using the indifference condition for a mixed strategy:

$$U_0(p_*) = U_0(0). \tag{26}$$

We can solve for $U_0(p_*)$ by combining equations (22) and (26), which gives us

$$U_0(p_*) = \frac{p_* - \mu\theta + \lambda_1 U_1(p_*)}{r + \lambda_1}. \tag{27}$$

Then, combining (23) with (27) we get

$$U_0(p_*) = \frac{p_*}{r} - \frac{\mu\theta}{r}\frac{r + \lambda_0}{r + \kappa} \tag{28}$$

$$U_1(p_*) = \frac{p_*}{r} - \frac{\mu\theta}{r}\frac{\lambda_0}{r + \kappa}. \tag{29}$$

The value of $p_*$ can thus be obtained from

$$U_0(p_*) = \frac{\lambda_1}{r + \lambda_1}\left[U_1(1) - c\right]. \tag{30}$$

The strategies above constitute an equilibrium as long as the following conditions are satisfied

1. $U_1(1) - c \geq 0$.
2. $U_1(p) \geq U_1(1) - c$ for $p \geq p_*$.
3. $U_0(p) \geq U_0(0)$ for $p > p_*$.

A necessary condition for the above strategy to be optimal is that $U_1(p) \geq U_1(1) - c$ for $p \geq p_*$. If $U_1$ is increasing in $p_*$, then this condition is satisfied if and only if

$$U_1(p_*) \geq U_1(1) - c = \left(1 + \frac{r}{\lambda_1}\right) U_0(p_*), \tag{31}$$

where we have used the equilibrium condition (30). Combining (28), (29) and (31) we get the following upper bound for the disclosure threshold $p_*$

$$p_* \leq \mu\theta.$$

The disclosure threshold is lower than the myopic threshold $\mu\theta$, which means that the manager delays bad news, relative to the case with permanent shocks. When the price reaches $\mu\theta$, the manager has the option to wait further and see if the asset recovers its value. If it does, the manager avoids the negative price consequences of bad news disclosures. Of course, this bet only makes sense if there is a positive probability of "resurrection" (i.e., $\lambda_1 > 0$). The idea that managers may withhold bad news hoping that the firm's financial standing will improve is borne out by the survey evidence in Graham, Harvey and Rajgopal (2005). Some CFOs claim that they delay bad news disclosures in the hope that they may never have to release the bad news if the firm's status improves. This is, in essence, Verrecchia (1983)'s alternative explanation.[25]

The condition $U_1(1) - c \geq 0$ is satisfied if and only if $U_0(p_*) \geq 0$. Thus, using (28) we obtain a lower bound for $p_*$ given by

$$p_* \geq \frac{r + \lambda_0}{r + \kappa}\mu\theta.$$

This lower bound reveals an intuitive feature of the model: if litigation costs are too high, the bound will hit 1 which means that no asymmetry of information can ever be experienced in equilibrium –negative information must be revealed immediately when litigation costs are prohibitively high.

**Proposition 6.** *Let $\hat{p}$ in equation (5) be the unique positive solution to $f(\hat{p}) = 0$, and let's define $\underline{p} := \max\big(\hat{p}, \mu\theta(r + \lambda_0)/(r + \kappa)\big)$. For any set of parameters $(\kappa, \bar{p}, \mu, \theta)$ such that $\hat{p} < \mu\theta$ and threshold $p_* \in (\underline{p}, \mu\theta)$, let $U_v(p)$ be the solution*

[25] Verrecchia (1983) argues "An alternative to my explanation for why a manager delays the reporting of 'bad news' is that he hopes that during the interim some 'good news' will occur to offset what he has to say.' The disadvantage of this explanation is that it ignores the fact that rational traders will correctly infer 'bad news' as soon as it becomes apparent that the information is being withheld"

*to equations* (20) *and* (21)*, with initial conditions* (28) *and* (29)*. Suppose that* $U_1(p)$ *satisfies*

$$U_0(p_*) = \frac{\lambda_1}{r+\lambda_1}\left[U_1(1) - c\right],$$

*where* $U_0(p_*)$ *is given by the initial condition* (28)*. Then, there exists an equilibrium such that*

1. *If* $V_t = 1$*, then* $d_t = \mathbf{1}_{\{p_{t-}=0\}}$*.*

2. *If* $V_t = 0$*, then:*

   - $d_t = \emptyset$ *for* $p_{t-} > p_*$*.*
   - *If* $p_{t-} = p_*$*, then the manager discloses with intensity*

   $$\zeta = \kappa \frac{p_* - \bar{p}}{p_*(1-p_*)} - \mu.$$

   - *If* $p_{t-} < p_*$*, then the manager discloses immediately with probability*

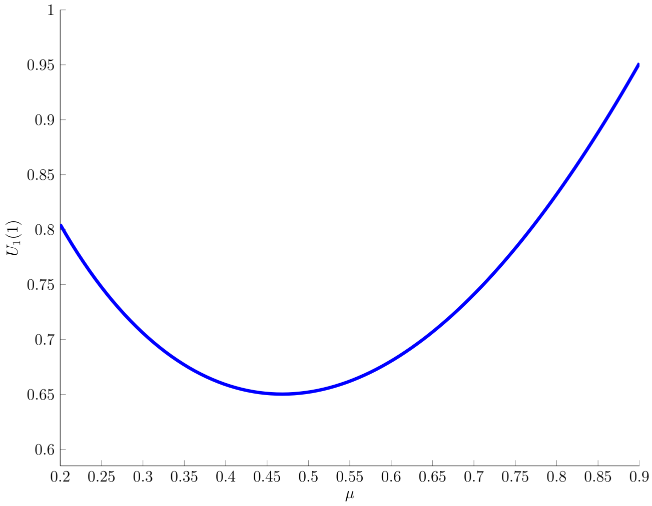
   $$\frac{p_{t-}}{1-p_{t-}} \frac{1-p_*}{p_*}.$$

# 5 Endogenous Monitoring

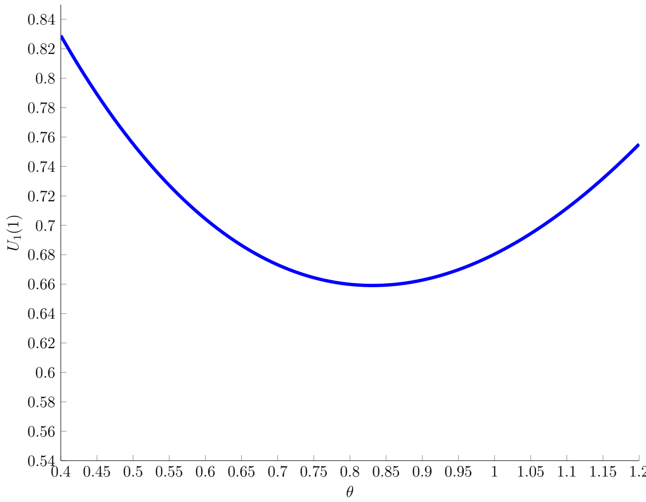
In this section we endogenize the monitoring intensity, $\mu$. For simplicity, we restrict attention to the case with permanent shocks, that is we assume $\lambda_1 = 0$.

Suppose there is a fact finder who can investigate whether the firm has concealed information and let $\mu_t$ be his monitoring strategy. If the fact finder proves that the firm is hiding adverse information he gets a payoff $b$. For example, the fact finder may be a law firm looking for litigation opportunities and $b = \omega\theta$ and $\omega \in (0, 1)$ be the fraction of the litigation proceeds the lawyer gets to retain if he proves the firm has concealed information.

The fact finder has a linear monitoring technology; in particular, he can choose any intensity $\mu_t \in [0, \bar{\mu}]$ at a cost $k\mu_t$. Let $\tau_\ell$ be the time at which




**(a)** Expected payoff high type, $U_1(1)$.




**(b)** Expected payoff high type, $U_1(1)$.

**Figure 3:** Effect of changes in the arrival rate of bad news, $\mu$, and litigation cost $\theta$ on manager's payoff. Baseline parameters: $r = 0.1$, $c = 0.5$, $\bar{p} = 0.1$, $\kappa = 1.2$, $\theta = 1$, $\mu = 0.2$, and $\theta = 1$.

the fact finder discovers that the firm withheld information and $\tau_d$ the date at which the firm discloses negative information. Then, the fact finder 's payoff is

$$E\left[e^{-r\tau_\ell\wedge\tau_d}b\mathbf{1}_{\tau_\ell<\tau_d} - \int_0^{\tau_\ell\wedge\tau_d} e^{-rt}k\mu_t dt\right] = \int_0^\infty e^{-rt-\int_0^t(\mu_s+\zeta_s)ds}\big(b(1-p_t)-k\big)\mu_t dt. \tag{32}$$

We can maximize (32) pointwise to get

$$\mu_t = \begin{cases} 0 & \text{if } p_t > p_m \\ \mu_m & \text{if } p_t = p_m \\ \bar{\mu} & \text{if } p_t < p_m \end{cases} \tag{33}$$

for some $\mu_m \in [0, \bar{\mu}]$ and $p_m := (b-k)/b$. Under this strategy, the manager's disclosure strategy is still myopic (as $\mu_t$ is non-decreasing in time). Hence, the manager discloses with positive probability if and only if $p_t = \mu_t\theta$. Because the manager would never disclose bad news if $\mu_t = 0$, the disclosure threshold,

$p_*$, must satisfy $p_* \leq p_m$.

We need to pin-down $\mu_m$ in (33) to complete the equilibrium characterization. Two cases must be considered. In the first case, $\mu_m = \bar{\mu}$ and in the second case $\mu_m \in (0, \bar{\mu})$. Using the threshold $p_m$ together with the equilibrium condition $p_m \geq p_*$ we get

$$1 - \frac{k}{b} \geq \mu_m \theta.$$

The first case happens if $p_m > p_*$ for all $\mu \in [0, \bar{\mu}]$, which is the case if and only if $1 - k/b > \bar{\mu}\theta$. The equilibrium in this case has a bang-bang solution with either zero or maximal monitoring. When this condition is not satisfied, the equilibrium involves an interior value of $\mu_m$ and $p_m = p_*$. The manager begins disclosing with positive probability exactly at the same time the fact finder starts monitoring the firm.

We pin-down the value of $\mu_m$ using the indifference condition

$$p_m = 1 - \frac{k}{b} = \mu_m \theta = p_*,$$

which yields the equilibrium monitoring intensity

$$\mu_m = \frac{1}{\theta}\left(1 - \frac{k}{b}\right).$$

We summarize the previous discussion in the following proposition.

**Proposition 7.** *Suppose that $b > k$. In the model with endogenous monitoring there is an equilibrium in which the fact finder finds that the firm is hiding information with mean arrival rate*

$$\mu_t = \mathbf{1}_{\{p_t \leq p_m\}} \mu_m \text{ with threshold } p_m = 1 - \frac{k}{b},$$

*where the monitoring intensity is $\mu_m = \bar{\mu}$ if*

$$\frac{1}{\theta}\left(1 - \frac{k}{b}\right) > \bar{\mu}$$

*and*

$$\mu_m = \frac{1}{\theta}\left(1 - \frac{k}{b}\right)$$

*otherwise. The manager's disclosure threshold is* $p_* = \mu_m\theta$ *and the disclosure mean arrival rate is*

$$\zeta = \kappa\frac{p_* - \bar{p}}{p_*(1-p_*)} - \mu_m.$$

Most of the comparative statics are intuitive. For example, when $b = \omega\theta$, the monitoring threshold is increasing in $\omega$ and $\theta$ and decreasing in $k$. The monitoring intensity is weakly increasing in $\omega$ and $\theta$. The monitoring intensity $\mu_m$ may increase or decrease in $\theta$. It increases if $k > \omega\theta/2$ and decreases otherwise. This ambiguity is explained by the effect of $\theta$ on the disclosure strategy. Increasing $\theta$ makes monitoring more profitable, but it also increases the probability that the manager will preempt litigation by disclosing bad news, thus reducing the probability that the fact finder gets rewarded. Note that with endogenous monitoring the manager must always hide information with positive probability, otherwise he would have no incentives to investigate.[26]

# 6 Patent Length and Disclosure

An important application of the previous ideas arises when the value of assets is modified by an event taking place at a known date in the future. Think for example of an invention whose patent expires at time $T$, followed by a reduction in the firm's profits.

For concreteness, consider the pharmaceutical industry. Assume the asset is a drug which will go off-patent in period $T$. After that period, some generic drugs will enter the market, so competition will increase and profits will go down. Before time $T$, the pharmaceutical company receives information, continuously, about the drug's side effects: for example the firm may learn about the increase in the probability of having a heart attack caused by the drug.[27]

[26] Rahman (2012) finds that optimal contracts that provide simultaneous incentives to agents and monitors necessary require some misbehavior (in equilibrium) by the agents.

[27] In the U.S., consumers, doctors, and lawyers report adverse incidents (e.g., the side

The pharmaceutical company has an incentive to delay and even conceal the adverse information because, if made public, the information would impact the demand for the drug, reducing both the manufacturer's expected revenues and stock price. Concealing the information is, however, risky from a legal standpoint and as before may trigger litigation costs.[28]

In the following, we study how the patent length affects the speed at which the firm will disclose adverse information. To model this situation suppose that the value of the asset in the high state is

$$V_1(t) = \begin{cases} \overline{V} - e^{-\rho(T-t)}(\overline{V} - \underline{V}) & \text{if } t \leq T \\ \underline{V} & \text{if } t > T. \end{cases} \tag{34}$$

Before expiration of the patent, the firm earns a flow of profits $\overline{v}$. After the patent expires, competition reduces the profits of the firm to $\underline{v}$. Under this specification, the value of the patent is given by (34), where $\overline{V} = \overline{v}/\rho$, $\underline{V} = \underline{v}/\rho$ and $\rho$ is the market's discount rate. As before, the value of the firm in the low state is $V_0(t) = 0$ for all $t \geq 0$. Because the benefit of not disclosing negative information is decreasing in time, the optimal disclosure strategy (of bad news) is given by a threshold $p_*(t)$ such that the low type manager discloses with positive probability only when $p_t = p_*(t)$. For the same reasons as before, the equilibrium must entail the use of mixed strategies. Let $\zeta_t > 0$ be the mean arrival rate of bad news disclosure. For $t < T$, the evolution of the value of

effects of a drug) to the company that manufactures the drug. Even though the company is obliged to disclose the information by resubmitting it, within two weeks, to the Food and Drugs Administration (FDA), in practice, the companies delay the resubmission of the information. For example, in the case of Avandia, the Times magazine asserts "Congressional reports revealed that GSK sat on early evidence of the heart risks of its drug, and that the FDA knew of the dangers months before it informed the public" (Times, August 12, 2010)

[28]The case of Glaxo's diabetes drug Avandia is paradigmatic. Its sales were \$2.5-billion in 2006; however, following a study published in the New England Journal of Medicine in 2007 that linked the drug's use to an increased risk of heart attack, sales plummeted to \$9.5-million in 2012. In 2012, the U.S. Justice Department announced GSK had agreed to plead guilty and pay a \$3 billion fine, in part for withholding the results of two studies of the cardiovascular safety of Avandia between 2001 and 2007 (New York Times, July 2, 2012).

the patent and beliefs evolves according to

$$\begin{aligned}\dot{V}_1(t) &= -\rho\big(\overline{V} - V_1(t)\big)\\ \dot{p}_t &= \kappa(\bar{p} - p_t) + (\mu + \zeta_t)p_t(1 - p_t).\end{aligned}$$

In principle, this may seem a very complicated problem given the non-stationarity of the setup. However, when $\lambda_1 = 0$, the decision of the low type manager to disclose his information is a monotone optimal stopping problem, which means that the myopic stopping rule is optimal (Ross, 1971). Accordingly, we have that the manager does not disclose negative information if $p_t V_1(t) > \mu\theta$ and discloses with positive probability if $p_t V_1(t) = \mu\theta$. Let us define $T_* := \inf\{t > 0 : p_t V_1(t) = \mu\theta\}$ and $T_{**} := \inf\{t > 0 : V_1(t) = \mu\theta\}$, where

$$T_{**} = T - \frac{1}{\rho}\ln\left(\frac{\overline{V} - \underline{V}}{\overline{V} - \mu\theta}\right)$$

For all $t \in (T_*, T_{**})$ the conditions $p_t V_1(t) = \mu\theta$ is satisfied. This means that $d(p_t V_1(t))/dt = 0$ so the equilibrium condition to determine $\zeta_t$ is

$$\dot{p}_t V_1(t) + p_t \dot{V}_1(t) = 0.$$

which reduces to

$$\kappa\frac{\bar{p}V_1(t)}{\mu\theta} + (\mu + \zeta_t)(1 - p_t) = \kappa + \rho\left(\frac{p_t\overline{V}}{\mu\theta} - 1\right).$$

Hence, we have that $\zeta_t$ is given by

$$\zeta_t = \frac{1}{1 - p_t}\left[\kappa - \rho + \frac{1}{\mu\theta}\big(\rho p_t\overline{V} - \kappa\bar{p}V_1(t)\big)\right] - \mu. \tag{35}$$

The equilibrium condition $p_t V_1(t) \geq \mu\theta$ can be satisfied for all $t \leq T$ if and only if $\underline{V} \geq \mu\theta$. Otherwise, there is $T_{**}$, as defined above, such that a low type manager discloses his private information with probability 1 when he is close to the expiration of the patent. Summarizing our previous discussion, the equilibrium disclosure strategy is:

$$\zeta_t = \begin{cases} 0 & \text{if } t \leq T_* \\ \frac{1}{1-p_t}\left[\kappa - \rho + \frac{1}{\mu\theta}\left(\rho p_t \overline{V} - \kappa \bar{p} V_1(t)\right)\right] - \mu & \text{if } T_* < t < T_{**} \\ \infty & \text{if } t \geq T_{**}, \end{cases} \tag{36}$$

where $\zeta_t = \infty$ indicates immediate disclosure. This means that, conditionally on not disclosing negative information, $p_t = 1$ for $t \geq T_*$. Figure 4 provides a numerical example showing the dynamics of disclosure in this case. Two elements are worth noting. First, disclosure tends to cluster around expiration. Furthermore, the longer the patent the lower the disclosure rate (i.e., the longer the disclosure delay). This intuitive result, which we believe is new to the literature, suggests that from a social point of view patent length should be shorter the more socially relevant the disclosures of the manufacturer are.

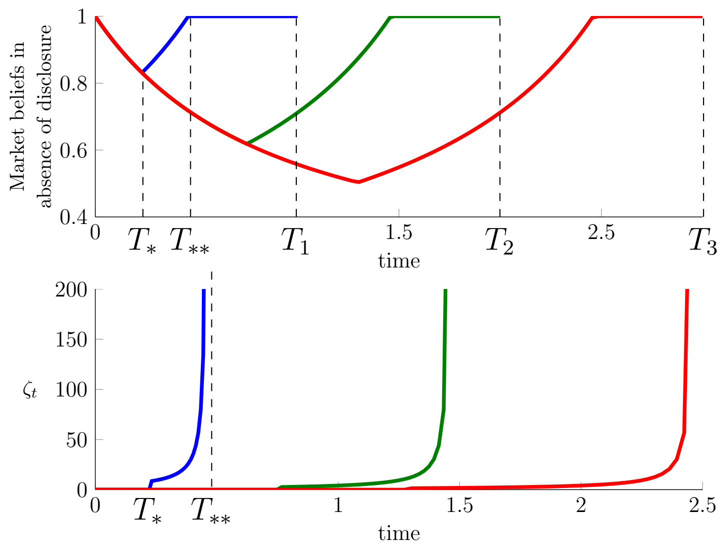



**Figure 4:** Evolution of beliefs in absence of disclosure and intensity of disclosure of negative information for different patent length. Here, $T_1 = 1$, $T_2 = 2$ and $T_3 = 3$ are three possible patent length $T$. $T_*$ and $T^*$ correspond to the thresholds for the case with patent length $T_1$. The value of $T_*$ and $T^*$ for $T_1, T_2$ and $T_3$ are $\mathbf{T}_* = (0.23, 0.74, 1.29)$ and $\mathbf{T}_{**} = (0.45, 1.45, 2.45)$, respectively.

# 7 Concluding Remarks

This paper studies a model of dynamic costly disclosure in which we incorporate the costs of delaying the disclosure of negative information, in particular litigation costs. The introduction of litigation costs introduces a signaling component to the disclosure environment. The market reacts positively to delays when they expect negative news to be disclosed. As we discuss in the introduction, the disclosure of bad news crowds out the disclosure of good news.

Our model is flexible enough to allow for many extensions and applications. For example, we analyze the effect that patent length has on the rate of disclosure of a monopolist. Thus, it provides a new potential cost associated with a strong patent that, to the best of our knowledge, is novel to the literature. The model is also flexible enough to incorporate endogenous monitoring by a rent seeking agent.

That being said, our model has several limitations. First, the state of nature is binary. One could consider the possibility of asset values that are continuously distributed. Second, we have modeled the public information as a Poisson process. An interesting but involved extension would be to consider a public information process that follows a Brownian motion whose drift depends on the state of nature, along the lines of the information structure considered by Daley and Green (2012). Again, this would allow for a more realistic characterization of stock returns.

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# Appendix (Not For Publication)

## A When The Cost of Disclosure Is Priced By the Market

The manager's objective function in equation (2) implicitly assumes that the cost of disclosure is borne by the manager. Our model can be extended to consider the cost of disclosure as a negative cash flow borne by the firm's shareholders. For simplicity we present a sketch of this extension only for the case without public news (that is, $\mu = 0$) and no litigation risk. Suppose that the manager compensation is a combination of the future stock prices and the firm's future cash flows, and suppose this combination is linear. Then, the manager's payoffs is

$$\mathcal{U}_t(d,\sigma) := E^d\left[\int_t^{\tau_M} e^{-\rho(s-t)}P_s ds + e^{-\rho\tau_M}V_{\tau_M} - C\sum_{t\le s<\tau_M} e^{-\rho(s-t)}\sigma_s\middle|\mathcal{F}_t, V_t\right],$$

which is equivalent up to some affine transformation to

$$\overline{\mathcal{U}}_t(d,\sigma) := p_t + E^d\left[\int_t^{\infty} e^{-r(s-t)}P_s ds - \eta C\sum_{t\le s} e^{-r(s-t)}\sigma_s\middle|\mathcal{F}_t, V_t\right],$$

where $\eta := (r+\kappa)/\gamma$. Therefore, the equilibrium is the same if we take the objective function of the manager to be

$$U_t(d,\sigma) := E^d\left[\int_t^{\infty} e^{-r(s-t)}P_s ds - \eta C\sum_{t\le s} e^{-r(s-t)}\sigma_s\middle|\mathcal{F}_t, V_t\right], \tag{37}$$

In section 2 we assumed that the cost of disclosure is borne by the manager, when this is the case, the share price is $P_t = E_t(V_{\tau_M})$. This is no longer true when the cost of disclosure is borne by shareholders. In this latter case, the price is

$$P_t = E^d\left[V_{\tau_M} - C\sum_{t\le s<\tau_M}\sigma_s\middle|\mathcal{F}_t\right] = \frac{\kappa}{\gamma+\kappa}\bar{p} + \frac{\gamma}{\gamma+\kappa}p_t - E^d\left[C\sum_{t\le s}e^{-\gamma s}\sigma_s\middle|\mathcal{F}_t\right] \tag{38}$$

Direct computation shows that[29]

$$\psi(p,p_*) := E^d\left[\sum_{t\le s} e^{-\gamma s}\sigma_s\Big|\mathcal{F}_t\right] = \frac{\left(\frac{\lambda}{\gamma+\lambda}+\frac{\gamma}{\gamma+\lambda}p_*\right)\left(\frac{p_*-\bar{p}}{p-\bar{p}}\right)^{\frac{\gamma}{\kappa}}}{1-\left(\frac{\lambda}{\gamma+\lambda}+\frac{\gamma}{\gamma+\lambda}p_*\right)\left(\frac{p_*-\bar{p}}{1-\bar{p}}\right)^{\frac{\gamma}{\kappa}}}.$$

Thus, the price is

$$P_t = q(p_t,p_*) = \frac{\kappa}{\gamma+\kappa}\bar{p} + \frac{\gamma}{\gamma+\kappa}p_t - \psi(p_t,p_*)C, \tag{39}$$

where we make the dependence of the price on the disclosure threshold $p_*$ explicit. We can write the manager's objective function as

$$U_t(d,\sigma) := E^d\left[\int_t^\infty e^{-r(s-t)}q(p_s,p_*)ds - \eta C\sum_{t\le s}e^{-r(s-t)}\sigma_s\Big|\mathcal{F}_t, V_t\right], \tag{40}$$

The value function satisfies the following HJB equations

$$\begin{aligned} rU_1(p) &= q(p,p_*) + \kappa(\bar{p}-p)U_1'(p) + \lambda_0[U_0(p)-U_1(p)] \\ rU_0(p) &= q(p,p_*) + \kappa(\bar{p}-p)U_0'(p) + \lambda_1[U_1(p)-U_0(p)] \end{aligned}$$

subject to the same boundary conditions as in the baseline case. We can solve the HJB equations following the same computations as the ones in appendix B. The value function $U$ is given by

$$\begin{aligned} U_0(p) &= U_1(p) - \frac{r}{r+\lambda_1}\left(\frac{p_*-\bar{p}}{p-\bar{p}}\right)^{1+\frac{r}{\kappa}}\Big(U_1(1)-\eta C\Big) \\ U_1(p) &= \int_0^{T(p,p_*)} e^{-rt}q\big(\phi_t(p),p_*\big)dt + \delta(p,p_*)\Big(U_1(1)-\eta C\Big), \end{aligned}$$

where

$$\delta(p,p_*) := \left(\frac{p_*-\bar{p}}{p-\bar{p}}\right)^{\frac{r}{\kappa}}\left[\frac{r\bar{p}+\kappa\bar{p}}{r+\kappa\bar{p}} + \frac{r(1-\bar{p})}{r+\kappa\bar{p}}\frac{p_*-\bar{p}}{p-\bar{p}}\right].$$

$q(p_t,p_*)$ and $q(p_*,p_*)$ are decreasing functions of $p_*$ ($\psi$ is increasing in $p_*$). This means that it is possible to replicate the steps in the proof of Propositions 3 and 4 to get an

[29] The derivation of $\psi$ uses the following recursive representation of $\psi$:

$$\psi(p,p_*) = e^{-\gamma T(p,p_*)}\left(p_*\psi(1,p_*) + \frac{\lambda_1}{\lambda_1+\gamma}\psi(1,p_*)\right) = e^{-\gamma T(p,p_*)}\Lambda(p_*)\psi(1,p_*),$$

where $T(p,p_*)$ is defined as $\inf\{t>0|p_t=p_*, p_0=p\}$.

analogous characterization of the equilibrium.

# B Proofs of Section 2

## Proof of Proposition 1

Let $\Delta(p) := U_1(p) - U_0(p)$, which satisfies

$$(r+\kappa)\Delta(p) = f(p)\Delta'(p) + \mu[U_0(p) - U_0(0)]. \tag{41}$$

Differentiating the HJB equation we get

$$rU_0'(p) = 1 + f'(p)U_0'(p) + f(p)U_0^{''}(p) + \lambda_1\Delta'(p) - \mu U_0'(p) \tag{42}$$
$$rU_1'(p) = 1 + f'(p)U_1'(p) + f(p)U_1^{''}(p) - \lambda_0\Delta'(p). \tag{43}$$

The proof is a direct consequence of the following two lemmas.

**Lemma 2.** *Suppose there is $p^1 \geq p_*$ such that $U_1'(p^1) = 0$, then $U_0'(p^1) > 0$.*

*Proof.* Evaluating (41) at $p_*$ we get that $U_0'(p_*) > 0$. If $U_0'$ is nondecreasing for $p > p_*$ we are done. Suppose that $U_0'(p)$ is decreasing for some $p > p_*$, then there must be some $p > p_*$ such that $U_0'(p) = 0$. Let $p^0 = \inf\{p \geq p_* : U_0'(p) < 0\}$. We have two possibilities, $p^0 \geq p^1$ or $p^0 < p^1$. Suppose that $p^0 < p^1$, if this is the case, using equation (42), we get $-f(p^0)U_0''(p^0) = 1 + \lambda_1 U_1'(p^0) > 0$. This means that $U_0''(p^0) > 0$ which contradicts the fact that $p^0 = \inf\{p > p_* : U_0'(p) < 0\}$ so it must be the case that $p^0 \geq p^1$. But then, by definition of $p^0$, we have $U_0'(p^1) > 0$. ∎

**Lemma 3.** *Suppose that $U_1'(p_*) \geq 0$, then $U_1$ is nondecreasing for all $p \geq p_*$.*

*Proof.* Suppose that $U_1(p)$ is decreasing in some interval, then there is $p$ such that $U_1'(p) = 0$. Let's define $p^1 = \inf\{p > p_* : U_1'(p) < 0\}$. Then, by (43) we have that

$$-f(p^1)U_1''(p^1) = 1 + \lambda_0 U_0'(p^1).$$

By lemma 2 we have $U_0'(p^1) > 0$. This means that $U_1''(p^1) > 0$ which is a contradiction with $p^1 = \inf\{p > p_* : U_1'(p) < 0\}$. ∎

*Proof Proposition 1.* From the boundary condition we have $U_1(p) = U_1(1) - c$; moreover, $U_1$ is nondecreasing by lemma 3. Hence, $U_1(p) \geq U_1(1) - c$ for all $p > p_*$. ∎

## Proof of Proposition 2

**Lemma 4.** *Let $p_*^1 < p_*^2$ be two equilibrium thresholds, then $U_1(1|p_*^1) > U_1(1|p_*^2)$.*

*Proof.* The solution to the HJB equation satisfies (Davis, 1993, Theorem 32.10, p. 94)

$$\begin{aligned}
U_1(1) &= E\left[\int_0^\infty e^{-rt}p_t dt - c\sum_{t\geq 0} e^{-rt}d_t\right] \\
&= E\left[\int_0^\infty e^{-rt}E(V_t\big|\mathcal{F}_t)dt - c\sum_{t\geq 0} e^{-rt}d_t\right] \\
&= \int_0^\infty e^{-rt}E(V_t)dt - cE\left[\sum_{t\geq 0} e^{-rt}d_t\right] \\
&= \int_0^\infty e^{-rt}E(V_t)dt - \frac{\delta}{1-\delta}c,
\end{aligned}$$

where $\delta := E(e^{-r\tau_d})$ and $\tau_d := \inf\{t \geq 0 : d_t = 1\}$. Let $\tau_d^1$ and $\tau_d^2$ be the first disclosure times for $p_*^1$ and $p_*^2$, respectively. To show that $U_1(1|p_*^1) > U_1(1|p_*^2)$ it is sufficient to show that $\tau_d^1 \geq \tau_d^2$ and that $\tau_d^1(\omega) > \tau_d^2(\omega)$ for a positive measure set of states $\omega$.

Let $\tau_N := \inf\{t \geq 0 : dN_t = 1\}$ and let $T_*^i$, $i = 1, 2$ be given by $\phi_{T_*^i} = p_*^i$ where $\phi_t$ is the solution to the differential equation

$$\frac{dp_t}{dt} = \kappa(\bar{p} - p_t) + \mu p_t(1 - p_t),\ p_0 = 1.$$

By construction we have $T_*^2 < T_*^1$. We consider several cases:

1. If $\tau_N < T_*^2$ then $\tau_d^2 = \tau_d^1$.
2. If $\tau_N > T_*^2$ and $V_{T_*^2} = 1$ then $\tau_d^2 = T_*^2 < \tau_d^1$.
3. If $\tau_N > T_*^2$ and $V_{T_*^2} = 0$ we have several sub-cases. Let $\sigma = \inf\{t > T_*^2 : V_t = 1\}$.
   (a) If $\tau_N < T_*^1$ then $\tau_d^2 = \tau_d^1 = \inf\{t \geq \tau_N : V_t = 1\}$.
   (b) If $\tau_N > T_*^1$ and $\sigma < T_*^1$ then $\tau_d^2 = \sigma < T_*^1 \leq \tau_d^1$.
   (c) If $\tau_N > T_*^1$ and $\sigma > T_*^1$ then $\tau_d^2 = \tau_d^1 = \sigma$.

According, $\tau_d^1 \geq \tau_d^2$ a.s. and $\Pr(\tau_d^1 > \tau_d^2) > 0$ which means that $E(e^{-r\tau_d^1}) < E(e^{-r\tau_d^2})$ and $U_1(1|p_*^1) > U_1(1|p_*^2)$. ∎

**Lemma 5.** *Suppose that $U_1'(p_*|p_*) \geq 0$ and $U_1(1|p_*) - c \geq 0$, then $U_1(p|p_*)$ is non increasing in $p_*$.*

*Proof.* Following the same computation as in (Davis, 1993, Theorem 32.10, p. 94) we can integrate the HJB equation to get

$$U_0(p_t) = \int_t^{T_*} e^{-(r+\lambda_1+\mu)(s-t)}\Big(p_s + \lambda_1 U_1(p_s) + \mu U_0(0)\Big)ds + e^{-(r+\lambda_1+\mu)(T_*-t)}U_0(0)$$
$$U_1(p_t) = \int_t^{T_*} e^{-(r+\lambda_0)(s-t)}\Big(p_s + \lambda_0 U_0(p_s)\Big)ds + e^{-(r+\lambda_0)(T_*-t)}[U_1(1) - c].$$

where $T_*$ is the time it gets for beliefs to reach $p_*$ in absence of any shock. Differentiating with respect to $p_*$ we get

$$\begin{aligned}\frac{\partial}{\partial p_*}U_0(p_t) &= \int_t^{T_*} e^{-(r+\lambda_1+\mu)(s-t)}\Big(\lambda_1 \frac{\partial}{\partial p_*}U_1(p_s) + \mu\frac{\partial}{\partial p_*}U_0(0)\Big)ds + e^{-(r+\lambda_1+\mu)(T_*-t)}\frac{\partial}{\partial p_*}U_0(0)\\ &\quad + \Big[e^{-(r+\lambda_1+\mu)(T_*-t)}\Big(p_* + \lambda_1 U_1(p_*) + \mu U_0(0)\Big) - (r+\lambda_1+\mu)e^{-(r+\lambda_1+\mu)(T_*-t)}U_0(p_*)\Big]\frac{\partial T_*}{\partial p_*}\\ \frac{\partial}{\partial p_*}U_1(p_t) &= \int_t^{T_*} e^{-(r+\lambda_0)(s-t)}\lambda_0\frac{\partial}{\partial p_*}U_0(p_s)ds + e^{-(r+\lambda_0)(T_*-t)}\frac{\partial}{\partial p_*}U_1(1)\\ &\quad + \Big[e^{-(r+\lambda_0)(T_*-t)}\Big(p_* + \lambda_0 U_0(p_*)\Big) - (r+\lambda_0)e^{-(r+\lambda_0)(T_*-t)}U_1(p_*)\Big]\frac{\partial T_*}{\partial p_*}\end{aligned}$$

Noting that $U_1(p_*) = U_1(1) - c$ and $U_0(p_*) = U_0(0) = \lambda_1[U_1(1) - c]/(r+\lambda_1)$

$$\begin{aligned}\frac{\partial}{\partial p_*}U_0(p_t) &= \int_t^{T_*} e^{-(r+\lambda_1+\mu)(s-t)}\Big(\lambda_1 \frac{\partial}{\partial p_*}U_1(p_s) + \mu\frac{\partial}{\partial p_*}U_0(0)\Big)ds + e^{-(r+\lambda_1+\mu)(T_*-t)}\frac{\partial}{\partial p_*}U_0(0)\\ &\quad + e^{-(r+\lambda_1+\mu)(T_*-t)}p_*\frac{\partial T_*}{\partial p_*} \qquad (44)\\ \frac{\partial}{\partial p_*}U_1(p_t) &= \int_t^{T_*} e^{-(r+\lambda_0)(s-t)}\lambda_0\frac{\partial}{\partial p_*}U_0(p_s)ds + e^{-(r+\lambda_0)(T_*-t)}\frac{\partial}{\partial p_*}U_1(1)\\ &\quad + e^{-(r+\lambda_0)(T_*-t)}\left[p_* - \frac{r(r+\lambda_1+\lambda_0)}{r+\lambda_1}U_1(p_*)\right]\frac{\partial T_*}{\partial p_*}. \qquad (45)\end{aligned}$$

Evaluating the HJB equation at $p_*$ we get

$$p_* - \frac{r(r+\lambda_1+\lambda_0)}{r+\lambda_1}U_1(p_*) = -f(p_*)U_1'(p_*)$$

which is greater than or equal to zero if $U_1'(p_*) \geq 0$. Evaluating (44) and (45) at $T_*$ we get

$$\frac{\partial}{\partial p_*}U_0(p_*) = \frac{\lambda_1}{r+\lambda_1}\frac{\partial}{\partial p_*}U_1(1) + p_*\frac{\partial T_*}{\partial p_*}$$
$$\frac{\partial}{\partial p_*}U_1(p_*) = \frac{\partial}{\partial p_*}U_1(1) + \left[p_* - \frac{r(r+\lambda_1+\lambda_0)}{r+\lambda_1}U_1(p_*)\right]\frac{\partial T_*}{\partial p_*}$$

Hence, using that $U_1(1)$ is decreasing in $p_*$ (Lemma 4) and $\partial T_*/\partial p_* < 0$, we get that $U_0(p_*) = U_0(0)$ and $U_1(p_*)$ are also decreasing in $p_*$. Then, by working backward from $t = T_*$, it is straightforward that (44) and (45) must be negative for all $t \leq T_*$ and hence for all $p \geq p_*$. ∎

**Lemma 6.** *Suppose that $U_1(1|p_*) - c \geq 0$, then $U_1'(p_*|p_*) = 0 \Rightarrow \frac{\partial}{\partial p_*}U_1'(p_*|p_*) > 0$.*

*Proof.* Rearranging the HJB equation we can write

$$U_1'(p|p_*) = \frac{rU_1(p|p_*) - p - \lambda_0[U_0\,(p|p_*) - U_1(p|p_*)]}{f(p)}$$

Evaluating at $p = p_*$ and using the boundary conditions, equations (??) and (??), yields

$$\begin{aligned} U_1'(p_*|p_*) &= \frac{rU_1(p_*|p_*) - p_* + U_1(p_*|p_*)\frac{r\lambda_0}{r+\lambda_1}}{f(p_*)} \\ &= \frac{\frac{r(r+\kappa)}{r+\lambda_1}U_1(p_*|p_*) - p_*}{f(p_*)} \end{aligned} \tag{46}$$

Now, we can show that

$$U_1'(p_*|p_*) = 0 \Rightarrow \frac{\partial}{\partial p_*}U_1'(p_*|p_*) > 0.$$

Differentiating equation (46) with respect to $p_*$ yields

$$\begin{aligned} \frac{\partial}{\partial p_*}U_1'(p_*|p_*) &= \frac{\frac{r(r+\kappa)}{r+\lambda_1}\frac{\partial U_1(p|p_*)}{\partial p_*}\Big|_{p=p_*} - 1}{f(p_*)} + \frac{f'(p_*)}{f(p_*)}U_1'(p_*|p_*) \\ &= \frac{\frac{r(r+\kappa)}{r+\lambda_1}\frac{\partial U_1(p|p_*)}{\partial p_*}\Big|_{p=p_*} - 1}{f(p_*)} > 0 \end{aligned}$$

But from Lemma 5 we know that $\frac{\partial U_1(p|p_*)}{\partial p_*} < 0$. This along with $f(p_*) < 0$ proves the lemma. ∎

*Proof of Proposition 2.* Suppose there exist $p_*^- < p_*^+$ such that

$$\begin{aligned} U_1'\left(p_*^-|p_*^-\right) &= 0 \\ U_1\left(1|p_*^+\right) - c &= 0. \end{aligned}$$

A direct consequence of Lemma 6 is that $U_1'\left(p_*|p_*\right)$ crosses 0 only once. Thus, $U_1'\left(p_*|p_*\right) \geq 0$ for $p_* \geq p_*^-$, and $U_1'\left(p_*|p_*\right) < 0$ for $p_* < p_*^-$. Moreover, from Lemma 5 we have that $U_1\left(p_*|p_*\right) - c \geq 0$ for all $p_* \leq p_*^+$. Hence, $p_*$ satisfies conditions (12) and (13) if and only if $p_* \in [p_*^-, p_*^+]$. ∎

# Proof of Proposition 3

*Proof.* We divide the proof of Proposition 3 in two steps. In step 1, we show that the functions in the proposition solve the HJB equation with the required boundary conditions. In step 2, we show that the solution constitutes an equilibrium.

## Step 1:

In the absence of any disclosure, the beliefs at time $t$ are given by

$$\phi_t(p_0) = \bar{p} + e^{-\kappa t}(p_0 - \bar{p}).$$

Let's define $T(p; p_*)$ as the time that it takes the beliefs to reach $p_*$ give that current beliefs are $p$. That is,

$$T(p; p_*) = -\frac{1}{\kappa}\log\left(\frac{p_* - \bar{p}}{p - \bar{p}}\right),$$

where $\frac{\partial T(p;p_*)}{\partial p} > 0$ and $\frac{\partial T(p;p_*)}{\partial p_*} < 0$. The results in Davis (1993, pp. 92-93) imply that the solution to the HJB equation (**??**)-(**??**) satisfies

$$\begin{aligned} U_0(p|p_*) = &\int_0^{T(p;p_*)} e^{-rt}\phi_t(p)dt + e^{-rT(p;p_*)}\Big[\Pr(V_{T(p;p_*)} = 0|V_0 = 0)U_0(p_*|p_*) \\ &+ \Pr(V_{T(p;p_*)} = 1|V_0 = 0)U_1(p_*|p_*)\Big] \\ U_1(p|p_*) = &\int_0^{T(p;p_*)} e^{-rt}\phi_t(p)dt + e^{-rT(p;p_*)}\Big[\Pr(V_{T(p;p_*)} = 0|V_0 = 1)U_0(p_*|p_*) \\ &+ \Pr(V_{T(p;p_*)} = 1|V_0 = 1)U_1(p_*|p_*)\Big]. \end{aligned}$$

Replacing $\Pr(V_{T(p;p_*)} = j|V_0 = i)$ for $i, j \in \{0, 1\}$, and using the boundary conditions, we can write the manager's expected payoff as

$$U_0(p|p_*) = \int_0^{T(p;p_*)} e^{-rt}\phi_t(p)dt + e^{-rT(p;p_*)}\left[\frac{r\bar{p}+\lambda_1}{r+\lambda_1} - \frac{r\bar{p}}{r+\lambda_1}e^{-\kappa T(p;p_*)}\right]\Big(U_1(1|p_*) - c\Big) \tag{47}$$

$$U_1(p|p_*) = \int_0^{T(p;p_*)} e^{-rt}\phi_t(p)dt + e^{-rT(p;p_*)}\left[\frac{r\bar{p}+\lambda_1}{r+\lambda_1} + \frac{r(1-\bar{p})}{r+\lambda_1}e^{-\kappa T(p;p_*)}\right]\Big(U_1(1|p_*) - c\Big). \tag{48}$$

Using equation (48) we can write $U_1(1|p_*)$ as

$$U_1(1|p_*,\kappa) = \frac{\int_0^{T(1;p_*)} e^{-rt}\phi_t(1)dt}{1-\delta(1)} - \frac{\delta(1)}{1-\delta(1)}c, \tag{49}$$

where

$$\delta(1) = e^{-rT(1;p_*)}\Big[\frac{r\bar{p}+\kappa\bar{p}}{r+\kappa\bar{p}} + \underbrace{\frac{r(p_*-\bar{p})}{r+\kappa\bar{p}}}_{\frac{r(1-\bar{p})}{r+\kappa\bar{p}}e^{-\kappa T(1;p_*)}}\Big].$$

The first term in (49) can be written as

$$U^{ND}(1)\frac{\overbrace{1-e^{-rT(1;p_*)}\frac{U^{ND}(p_*)}{U^{ND}(1)}}^{1-\delta(1)}}{1-\delta(1)} = U^{ND}(1).$$

Hence,

$$U_1(1|p_*) = U^{ND}(1) - \frac{\delta(1)}{1-\delta(1)}c, \tag{50}$$

## Step 2:

The only step left is to show that (12) and (13) imply $U_1(p) \geq U_1(1) - c$ for all $p > p_*$ so a threshold policy is optimal. We first show that (12) and (13) imply $U_1'(p) \geq 0$ for all $p > p_*$. The derivative of $U_1$ is given by

$$U_1'(p) = e^{-rT(p;p_*)}\Phi(p)\frac{\partial T(p;p_*)}{\partial p} + \int_0^{T(p;p_*)} e^{-(r+\kappa)t}dt \tag{51}$$

where

$$\Phi(p) := p_* - re^{-rT(p;p_*)}\left[\frac{r\bar{p}+\lambda_1}{r+\lambda_1} + \frac{(1-\bar{p})(r+\kappa)}{r+\lambda_1}e^{-\kappa T(p;p_*)}\right]\Big(U_1(1) - c\Big).$$

From here we get that $U_1'(p_*) \geq 0$ if and only if $\Phi(p_*) \geq 0$. Moreover, $U_1(1) - c > 0$ implies $\Phi'(p) > 0$, which means that $\Phi(p) \geq 0$ for all $p > p_*$. Accordingly, $U_1'(p) \geq 0$ for all $p > p_*$, and

$$U_1(p) = U_1(p_*) + \int_{p_*}^{p} U_1'(y)dy = U_1(1) - c + \int_{p_*}^{p} U_1'(y)dy > U_1(1) - c.$$

∎

# Proof of Proposition 4

*Proof of Proposition 4.* We show that if the cost of disclosure satisfies the conditions in the proposition then exist $p_*^-, p_*^+ \in (\bar{p}, 1)$ with the required properties.

**Claim 1:** If $c < \frac{r+\lambda_1}{r(r+\kappa)}$, then there is a threshold $p_*^+ \in (\bar{p}, 1)$ such that $U_1\left(1|p_*^+\right) - c = 0$.

First, from equation (48) we have that $U(1|\bar{p}) = U^{ND}(1)$. Hence, $U(1|\bar{p}) - c > 0$ if and only if

$$c < \frac{\lambda_1 + r}{r(r+\kappa)}.$$

Second, $U(1|1-\epsilon) - c < 0$ for $\epsilon$ close to zero. Let

$$\beta(\epsilon) := e^{-rT(1;1-\epsilon)} \left[ \frac{r\bar{p} + \lambda_1}{r + \lambda_1} + \frac{r(1-\bar{p})}{r+\lambda_1} e^{-\kappa T(1;1-\epsilon)} \right].$$

Using equation (48) we get that

$$(1 - \beta(\epsilon))\big[U(1|1-\epsilon) - c\big] < T(1; 1-\epsilon) - \beta(\epsilon)c,$$

where $T(1; 1-\epsilon) - \beta(\epsilon)c <$ for $\epsilon$ close to zero. Hence, by continuity there exist $p_*^+ \in (\bar{p}, 1)$ such that $U_1\left(1|p_*^+\right) - c = 0$. Moreover, equation (51) implies $U_1'\left(p_*^+|p_*^+\right) > 0$.

**Claim 2:** If $c < \frac{r+\lambda_1}{r(r+\kappa)}(1-\bar{p})$, then there there is $p_*^- < p_*^+$ such that $U'(p_*^-|p_*^-) = 0$.

First, we verify that $\lim_{p_* \downarrow \bar{p}} U_1'(p_*|p_*) < 0$. Using the HJB equation

$$U_1'(p_*|p_*) = \frac{\frac{r(r+\kappa)}{r+\lambda_1} U_1(p_*|p_*) - p_*}{\kappa(\bar{p} - p_*)}$$

Noting that $\lim_{p_* \downarrow \bar{p}} U_1(p_*|p_*) = U_1^{ND}(1) - c$, it suffices to show that

$$\frac{r(r+\kappa)}{r+\lambda_1}\big(U_1^{ND}(1) - c\big) - \bar{p} > 0,$$

which, after straightforward algebra, is satisfied if and only if $c < \frac{r+\lambda_1}{r(r+\kappa)}(1-\bar{p})$. Second, we verify that $\lim_{p_* \uparrow 1} U_1'(p_*|p_*) > 0$. When $p_* \uparrow 1$ the firm starts disclosing infinitely often. Hence, the cost of disclosure grows without bound. Moreover, the benefit of disclosing is bounded. Accordingly

$$\lim_{p_*\uparrow 1} \frac{r(r+\kappa)}{r+\lambda_1} U_1(p_*|p_*) - p_* < 0$$

so, from the HJB equation, $\lim_{p_*\uparrow 1} U_1'(p_*|p_*) > 0$. By continuity there is $p_*^- \in (\bar{p}, 1)$ with the required properties. Moreover, $U_1'(p_*^+|p_*^+) > 0$ implies that $p_*^- < p_*^+$.

∎

# C Proofs of Section 4

## C.1 Permanent Shocks

### C.1.1 Equilibrium with disclosure of good news

The value functions obey

$$rU_1(p) = p + f(p)U_1'(p) + \lambda_0[U_0(p) - U_1(p)] \tag{52}$$

$$rU_0(p) = p - \mu\theta + f(p)U_0'(p) - \mu U_0(p) \tag{53}$$

With boundary conditions

$$U_1(p) = U_1(1) - c, \text{ for all } p \leq p_* \tag{54}$$

$$U_0(p) = 0, \text{ for } p \leq p_* \tag{55}$$

**Lemma 7.** *The solution to the HJB equation is*

$$U_0(p) = \int_0^{T(p)} e^{-(r+\mu)t}\big(\phi_t(p) - \mu\theta\big)dt$$

$$U_1(p) = \int_0^{T(p)} e^{-(r+\lambda_0)t}\left(\frac{\mu - \lambda_0 e^{-(\mu-\lambda_0)t}}{\mu-\lambda_0}\phi_t(p) - \frac{1-e^{-(r+\mu)(T(p)-t)}}{r+\mu}\lambda_0\mu\theta\right)dt + e^{-(r+\lambda_0)T(p)}\big(U_1(1)-c\big).$$

$$U_1(p) = U_1(1) - c, \textit{ for all } p \leq p_* \tag{56}$$

$$U_0(p) = 0, \textit{ for } p \leq p_* \tag{57}$$

*Proof.*

$$U_1(p) = \int_0^{T(p)} e^{-(r+\lambda_0)t} \left(\phi_t(p) + \lambda_0 U_0(\phi_t(p))\right) dt + e^{-(r+\lambda_0)T(p)}\big(U_1(1) - c\big)$$
$$= \int_0^{T(p)} e^{-(r+\lambda_0)t} \left(\phi_t(p) + \lambda_0 \int_t^{T(p)} e^{-(r+\mu)(s-t)}\big(\phi_s(p) - \mu\theta\big)ds\right) dt + e^{-(r+\lambda_0)T(p)}\big(U_1(1) - c\big)$$

The term accompanying $\mu\theta$ follows by simple integration. For the term accompanying $\phi_t(p)$ we change the order of integration to get

$$\begin{aligned}\int_0^{T(p)} \int_t^{T(p)} e^{-(r+\lambda_0)t} e^{-(r+\mu)(s-t)}\phi_s(p)dsdt &= \int_0^{T(p)} \int_t^{T(p)} e^{(\mu-\lambda_0)t} e^{-(r+\mu)s}\phi_s(p)dsdt \\ &= \int_0^{T(p)} e^{-(r+\mu)s}\phi_s(p) \int_0^{s} e^{(\mu-\lambda_0)t}dtds \\ &= \int_0^{T(p)} e^{-(r+\lambda_0)s}\phi_s(p)\frac{1-e^{-(\mu-\lambda_0)s}}{\mu-\lambda_0}ds.\end{aligned}$$

Hence,

$$\int_0^{T(p)} e^{-(r+\lambda_0)t} \left(\phi_t(p) + \lambda_0 \int_t^{T(p)} e^{-(r+\mu)(s-t)}\phi_s(p)ds\right) dt = \int_0^{T(p)} e^{-(r+\lambda_0)t}\frac{\mu-\lambda_0 e^{-(\mu-\lambda_0)t}}{\mu-\lambda_0}\phi_t(p)dt$$

∎

**Lemma 8.** *Assume $U_1'(p_*|p_*) \geq 0$ and $U_1(1|p_*) - c \geq 0$, then $\frac{\partial}{\partial p_*}U(1|p_*) < 0$, for all $p_* \in [\mu\theta, 1]$.*

*Proof of Lemma 8.* We first note that $\frac{\mu-\lambda_0 e^{-(\mu-\lambda_0)t}}{\mu-\lambda_0}\phi_t(1) = 1$. Hence, using Lemma 14 the two hypotheses become

$$U_1\left(1|p_*\right) - c = \int_0^{T} \frac{e^{-(r+\lambda_0)t}}{1-e^{-T(r+\lambda_0)}}\left(1 - \frac{1-e^{-(r+\mu)(T-t)}}{r+\mu}\lambda_0\mu\theta\right)dt - \frac{c}{1-e^{-(r+\lambda_0)T}} \geq 0, \tag{58}$$

and

$$U_1'(p_*|p_*) = \frac{(r+\lambda_0)\left(U_1\left(1|p^*\right)-c\right)-p^*}{f\left(p^*\right)} \geq 0 \Rightarrow (r+\lambda_0)\left(U_1\left(1|p^*\right)-c\right)-p^* \leq 0 \tag{59}$$

where $T = \frac{\ln\frac{\lambda_0-\mu+p_*\mu}{p_*\lambda_0}}{\lambda_0-\mu}$, or equivalently

$$p_* = \frac{\lambda_0-\mu}{e^{T(\lambda_0-\mu)}\lambda_0-\mu} \tag{60}$$

Now differentiating (58) with respect to $T$ yields

$$\begin{aligned}\frac{\partial\left(U_1\left(1|p_*\right)-c\right)}{\partial T} &= -\left(r+\lambda_0\right)\frac{e^{-T(r+\lambda_0)}}{1-e^{-T(r+\lambda_0)}}\left(U_1\left(1|p_*\right)-c\right)\\ &\quad+\frac{e^{-(r+\lambda_0)T}-\int_0^T e^{-(r+\lambda_0)t-(r+\mu)(T-t)}\lambda_0\mu\theta dt}{1-e^{-T(r+\lambda_0)}}\\ &\Rightarrow \frac{\partial\left(U_1\left(1|p_*\right)-c\right)}{\partial T}\left(\frac{e^{-T(r+\lambda_0)}}{1-e^{-T(r+\lambda_0)}}\right)^{-1}=-\left(r+\lambda_0\right)\left(U_1\left(1|p_*\right)-c\right)\\ &\quad+1-\int_0^T\frac{e^{-(r+\lambda_0)t-(r+\mu)(T-t)}}{e^{-(r+\lambda_0)T}}\lambda_0\mu\theta dt\\ &\geq -p_*+1-\int_0^T\frac{e^{-(r+\lambda_0)t-(r+\mu)(T-t)}}{e^{-(r+\lambda_0)T}}\lambda_0\mu\theta dt\end{aligned}$$

where the last inequality follows from (59) . Now, replacing $p_*$ by (60), yields

$$\begin{aligned}\frac{\partial\left(U_1\left(1|p_*\right)-c\right)}{\partial T}\left(\frac{e^{-T(r+\lambda_0)}}{1-e^{-T(r+\lambda_0)}}\right)^{-1} &\geq 1-\frac{\lambda_0-\mu}{e^{T\lambda_0-\mu T}\lambda_0-\mu}\\ &\quad-\int_0^T\frac{e^{-(r+\lambda_0)t-(r+\mu)(T-t)}}{e^{-(r+\lambda_0)T}}\lambda_0\mu\theta dt\\ &= \lambda_0\left(1-e^{T(\lambda_0-\mu)}\right)\frac{\mu\theta\left(e^{T(\lambda_0-\mu)}\lambda_0-\mu\right)-(\lambda_0-\mu)}{\left(e^{T(\lambda_0-\mu)}\lambda_0-\mu\right)(\lambda_0-\mu)}\end{aligned}$$

Now notice that

$$\lambda_0\left(1-e^{T(\lambda_0-\mu)}\right)\frac{\mu\theta\left(e^{T(\lambda_0-\mu)}\lambda_0-\mu\right)-(\lambda_0-\mu)}{\left(e^{T(\lambda_0-\mu)}\lambda_0-\mu\right)(\lambda_0-\mu)}=0\Leftrightarrow T=0 \text{ or } T=\frac{\ln\frac{\lambda_0-\mu+\mu^2\theta}{\lambda_0\mu\theta}}{\lambda_0-\mu}.$$

and

$$\lim_{T\to 0}\frac{\partial\left[\lambda_0\left(1-e^{T(\lambda_0-\mu)}\right)\frac{\mu\theta\left(e^{T(\lambda_0-\mu)}\lambda_0-\mu\right)-(\lambda_0-\mu)}{\left(e^{T(\lambda_0-\mu)}\lambda_0-\mu\right)(\lambda_0-\mu)}\right]}{\partial T}=\lambda_0\left(1-\mu\theta\right)>0$$

This means that $\frac{\partial(U_1(1|p_*)-c)}{\partial T}$ is positive for all $T\in[0,\frac{\ln\frac{\lambda_0-\mu+\mu^2\theta}{\lambda_0\mu\theta}}{\lambda_0-\mu}]$. This implies that $\frac{\partial(U_1(1|p_*)-c)}{\partial p_*}$ is negative for all $p_*\in[\frac{\mu\theta}{1+\mu\theta},1]\supsetneqq[\mu\theta,1]$. ∎

**Lemma 9.** *Suppose that $U_1'\left(p_*|p_*\right)\geq 0$ and $U_1(1|p_*)-c\geq 0$, then $U_v\left(p|p_*\right)$ is non increasing in $p_*$.*

*Proof of Lemma 9.* This follows directly upon adapting the proof of Lemma 9. ∎

**Lemma 10.** *Suppose that $U_1\left(1|p_*\right)-c\geq 0$, then $U_1'\left(p_*|p_*\right)=0\Rightarrow\frac{\partial}{\partial p_*}U_1'\left(p_*|p_*\right)>0$.*

*Proof of Lemma 10.* This follows directly from adapting the proof of Lemma 10. ∎

**Lemma 11.** *In any equilibrium with good news disclosure, the disclosure threshold must be greater or equal than $\mu\theta$.*

*Proof of Lemma 11.* Suppose there is an equilibrium with disclosure threshold $p_* < \mu\theta$. The value function of the low type is

$$U_0(p) = \int_0^{T(p_*)} e^{-(r+\mu)t}\big(\phi_t(p) - \mu\theta\big)dt,$$

so for $p < p_*$ we have $U_0(p) < 0$. This means that the low type would have incentives to disclose its type, which contradicts the fact that this is an equilibrium with disclosure of good news. ∎

**Lemma 12.** *Suppose there exist $p_*^- \leq p_*^+$, such that*

$$\begin{aligned} U_1'\left(p_*^-|p_*^-\right) &= 0 \\ U_1'\left(1|p_*^-\right) - c &= 0 \\ p_*^+ &\geq \mu\theta \end{aligned}$$

then $p_*$ is an equilibrium threshold if and only if $p_* \in [\max\left(p_*^-, \mu\theta\right), p_*^+]$.

*Proof of Lemma 12.* This follows directly after slightly adapting the proof of Proposition 1 and using Lemma 11 ∎

**Lemma 13.** *(i) A necessary and sufficient condition for the existence of equilibria with good news is that $c \leq \overline{c}$ where*

$$U_1\left(1|\mu\theta\right) - \overline{c} = 0.$$

*Also there is $\underline{c}$ such that*

$$\begin{aligned} p_*^- &= \mu\theta \\ U_1'\left(p_*^-|p_*^-\right) &= 0. \end{aligned}$$

*where*

$$\begin{aligned} \underline{c} &= (1 - e^{-(r+\lambda_0)T})\left(\frac{r + \mu - (r + \mu + \lambda_0)\,\mu\theta}{(r+\mu)\,(r+\lambda_0)} + \frac{\lambda_0\mu\theta}{(r+\mu)\,(\lambda_0 - \mu)}\frac{1 - e^{(\lambda_0-\mu)T}}{1 - e^{(r+\lambda_0)T}}\right) \\ \overline{c} &= \frac{e^{\left(\ln\frac{\lambda_0 - \mu + \mu^2\theta}{\lambda_0\mu\theta}\right)\frac{r+\lambda_0}{\lambda_0-\mu}}\left(r + \mu - \lambda_0\mu\theta\right) - \left(\mu\theta r + \mu - \lambda_0\right)}{(r+\lambda_0)\,(r+\mu)} e^{\left(\ln\frac{\lambda_0\mu\theta}{(\lambda_0-\mu+\mu^2\theta)}\right)\frac{r+\lambda_0}{\lambda_0-\mu}}. \end{aligned}$$

*Proof of Lemma 13.* Observe that

$$U_1\left(1|p_*\right) - c = 0 \Rightarrow U_1'\left(p_*|p_*\right) \geq 0.$$

Also

$$U_1'\left(p_*|p_*\right) \leq 0 \Rightarrow U_1\left(1|p_*\right) - c \geq 0.$$

Now recall that $p_*^+$ is defined by $U_1\left(1|p_*^+\right) - c = 0$, then from Lemma 8 and the Implicit Function Theorem we have that $\frac{\partial p_*^+}{\partial c} < 0$. Defining $\overline{c}$ by

$$U_1\left(1|\mu\theta\right) - \overline{c} = 0.$$

we have that $p_*^+ \geq \mu\theta$ for all $c \leq \overline{c}$. Moreover, the fact that $U_1'\left(p_*^+|p_*^+\right) > 0$ implies that either $U_1'(p_*|p_*) > 0$ in the interval $[\mu\theta, p_*^+]$ or it crosses zero at most once and from below at some $p_*^- \leq p_*^+$. Hence for any $c < \overline{c}$ there must be a unique interval of equilibrium thresholds: $[\min\left(\mu\theta, p_*^-\right), p_*^+]$. The value of $\overline{c}$ can be computed as

$$\overline{c} = \frac{e^{\left(\ln\frac{\lambda_0-\mu+\mu^2\theta}{\lambda_0\mu\theta}\right)\frac{r+\lambda_0}{\lambda_0-\mu}}\left(r+\mu-\lambda_0\mu\theta\right) - \left(\mu\theta r+\mu-\lambda_0\right)}{\left(r+\lambda_0\right)\left(r+\mu\right)} e^{\left(\ln\frac{\lambda_0\mu\theta}{\left(\lambda_0-\mu+\mu^2\theta\right)}\right)\frac{r+\lambda_0}{\lambda_0-\mu}}.$$

Finally, we denote by $\underline{c}$ the value of $c$ such that $p_*^- = \mu\theta$, where

$$U_1'\left(\mu\theta|\mu\theta\right) = 0$$

Using the HJB equation, we can verify that

$$U_1'(\mu\theta|\mu\theta) = 0 \implies U_1(\mu\theta|\mu\theta) = U_1(1|\mu\theta) - \underline{c} = \frac{\mu\theta}{r+\lambda_0},$$

where

$$U_1(1|\mu\theta) = \int_0^{T(\mu\theta)} e^{-(r+\lambda_0)t}\left(\frac{\mu-\lambda_0 e^{-(\mu-\lambda_0)t}}{\mu-\lambda_0}\phi_t(1) - \frac{1-e^{-(r+\mu)(T(p)-t)}}{r+\mu}\lambda_0\mu\theta\right)dt + e^{-(r+\lambda_0)T(\mu\theta)}\frac{\mu\theta}{r+\lambda_0}$$

The value of $\underline{c}$ can thus be computed as

$$\underline{c} = \left(1-e^{-(r+\lambda_0)T}\right)\left(\frac{r+\mu-\left(r+\mu+\lambda_0\right)\mu\theta}{\left(r+\mu\right)\left(r+\lambda_0\right)} + \frac{\lambda_0\mu\theta}{\left(r+\mu\right)\left(\lambda_0-\mu\right)}\frac{1-e^{(\lambda_0-\mu)T}}{1-e^{(r+\lambda_0)T}}\right)$$

where $T = \frac{\ln\frac{\lambda_0-\mu+\mu^2\theta}{\mu\theta\lambda_0}}{\lambda_0-\mu}$. ∎

### C.1.2 Equilibrium with disclosure of bad news

The value functions obey

$$rU_1(p) = p + f(p)U_1'(p) + \lambda_0[U_0(p) - U_1(p)] \tag{61}$$
$$rU_0(p) = p - \mu\theta + f(p)U_0'(p) - \mu U_0(p) \tag{62}$$

With boundary conditions

$$U_1(p_*) = \frac{\mu\theta}{r+\lambda_0}, \text{ for all } p \leq p_* \tag{63}$$
$$U_0(p) = 0, \text{ for } p \leq p_* \tag{64}$$

The time that it takes to reach the threshold is

$$T = \frac{\ln \frac{\lambda_0 - \mu + \theta\mu^2}{\mu\theta\lambda_0}}{\lambda_0 - \mu}.$$

The value functions can be written as

$$\begin{aligned} U_1(p) &= \int_0^{T(p_t)} e^{-s(r+\lambda_0)}\left(\phi_s(p) + \lambda_0 U_0(\phi_s(p))\right) ds + \frac{\mu\theta}{r+\lambda_0} e^{-T(p)(r+\lambda_0)} \\ U_0(p) &= \int_0^{T(p_t)} e^{-s(r+\mu)}\left(\phi_s(p) - \mu\theta\right) ds \end{aligned}$$

**Lemma 14.** *The solution to the HJB equation is*

$$U_0(p) = \int_0^{T(p)} e^{-(r+\mu)t}\big(\phi_t(p) - \mu\theta\big)dt$$
$$U_1(p) = \int_0^{T(p)} e^{-(r+\lambda_0)t}\left(\frac{\mu - \lambda_0 e^{-(\mu-\lambda_0)t}}{\mu - \lambda_0}\phi_t(p) - \frac{1 - e^{-(r+\mu)(T(p)-t)}}{r+\mu}\lambda_0\mu\theta\right) dt + e^{-(r+\lambda_0)T(p)}\frac{\mu\theta}{r+\lambda_0}.$$

$$U_1(p) = U_1(1) - c, \text{ for all } p \leq p_* \tag{65}$$
$$U_0(p) = 0, \text{ for } p \leq p_* \tag{66}$$

*Proof.* Identical to Lemma 7. ∎

**Lemma 15.** *There is an equilibrium with bad news if and only if $c \geq \hat{c}$ where $\hat{c}$ is defined by*

$$U_1(1) - \hat{c} = U_1(\mu\theta) = \frac{\mu\theta}{r+\lambda_0}, \tag{67}$$

*and the value of $\hat{c}$ has been computed at the end of the proof of Lemma 13 as $\underline{c}$.*

*Proof of Lemma 15.* A necessary and sufficient condition for an equilibrium with disclosure of bad news is that the high type does not disclose its type when $p_t = \mu\theta$. This happens if and only if

$$U_1(1) - c \leq \frac{\mu\theta}{r + \lambda_0}. \tag{68}$$

Noting that $U_1(1) = \mathcal{A}(1, \mu\theta) + \delta \frac{p_*}{r+\lambda_0}$, where $\mathcal{A}$ and $\delta$ are as in the proof of Lemma 16, does not depend on $c$ we can conclude from (68) that an equilibrium with disclosure of good news exists if and only if

$$c \geq \hat{c} := U_1(1) - \frac{\mu\theta}{r + \lambda_0}. \tag{69}$$

Thus, the definition of $\hat{c}$ in equation (69) coincides with $\underline{c}$ computed at the end of Lemma 13. ∎

## C.2 Comparing equilibria with good and bad news

**Lemma 16.** *Let $U_v^g(p)$ and $U_v^b(p)$ be the value function in an equilibrium with disclosure of good news and bad news with disclosure threshold $p_* = \mu\theta$, respectively. Then, for all $p$ and for all $v \in \{0, 1\}$ we have that $U_v^b(p) \geq U_v^g(p)$.*

*Proof.* There is nothing to prove for the low type as

$$U_0^g(p) = U_0^b(p) = \int_0^{T(p)} e^{-(r+\mu)t}\big(\phi_t(p) - \mu\theta\big)dt.$$

Let's define

$$\begin{aligned}
\mathcal{A}(p, p_*) &= \int_{T(p)}^{T(p_*)} e^{-(r+\lambda_0)t}\left(\frac{\mu - \lambda_0 e^{-(\mu-\lambda_0)t}}{\mu - \lambda_0}\phi_t(p) - \frac{1 - e^{-(r+\mu)(T(p)-t)}}{r + \mu}\lambda_0\mu\theta\right) dt \\
\delta(p, p_*) &= e^{-(r+\lambda_0)(T(p_*)-T(p))} \\
\delta &= \delta(1, \mu\theta).
\end{aligned}$$

We have the following equilibrium condition for an equilibrium with disclosure of bad news.

$$U_1^b(1) - c = \frac{\mathcal{A}(1, \mu\theta) - c}{1 - \delta} \leq \frac{\mu\theta}{r + \lambda_0}. \tag{70}$$

Similarly, we have that

$$U_1^g(p) = \frac{\mathcal{A}(1, \mu\theta) - c}{1 - \delta}.$$

Hence,

$$U_1^b(p) - U_1^g(p) = \delta(p, \mu\theta)\left[\frac{\mu\theta}{r + \lambda_0} - \big(U_1^g(1) - c\big)\right] = \delta(p, \mu\theta)\left[\frac{\mu\theta}{r + \lambda_0} - \frac{\mathcal{A}(1, \mu\theta) - c}{1 - \delta}\right] \geq 0,$$

where in the last inequality we use (70) ∎

**Lemma 17.** *Let's define*

$$\underline{c} := (1 - e^{-(r+\lambda_0)T}) \left( \frac{r + \mu - (r + \mu + \lambda_0)\,\mu\theta}{(r + \mu)\,(r + \lambda_0)} + \frac{\lambda_0 \mu\theta}{(r + \mu)\,(\lambda_0 - \mu)} \frac{1 - e^{(\lambda_0 - \mu)T}}{1 - e^{(r+\lambda_0)T}} \right).$$

*where* $T = T(\mu\theta) = \frac{\ln \frac{\lambda_0 - \mu + \mu^2\theta}{\mu\theta\lambda_0}}{\lambda_0 - \mu}$. *Then*

1. *If $c < \underline{c}$, then any equilibria with good news has a threshold strictly greater than $\mu\theta$. In particular, $p_*^- > \mu\theta$.*
2. *If $c < \underline{c}$, then the Pareto dominating equilibrium is the equilibrium with disclosure of good news and threshold $p_*^- > \mu\theta$. Alternatively, if $c \geq \underline{c}$ then the Pareto dominating equilibrium is the equilibrium with disclosure of bad news with threshold $\mu\theta$.*

*Proof.* Let $U_1^g(p|p_*)$ be the value function in an equilibrium with disclosure of good news with threshold $p_*$. Let also $p_*^-(c)$ be the threshold satisfying the smooth pasting condition $U_1^{g\prime}(p_*|p_*) = 0$ for a cost of disclosure $c$. From the proof of Lemma 13 we have that $p_*^-(\underline{c}) = \mu\theta$ and $U_1^g\big(1|p_*^-(c)\big) - c = \frac{p_*^-(c)}{r+\lambda_0}$. Using the implicit function theorem we get

$$\frac{d}{dc} p_*^-(c) = -\frac{r + \lambda_0}{1 - (r + \lambda_0)\partial U_1^g\big(1|p_*^-(c)\big)/\partial p_*} < 0,$$

where we have used that $\partial U_1^g\big(1|p_*^-(c)\big)/\partial p_* < 0$ (Lemma 8). Hence, the result in 1. follows from $p_*^-(\underline{c}) = \mu\theta$.

For 2. observe that a bad news equilibrium does not exist if $c < \underline{c}$ (Lemma 15). Hence, we only need to consider equilibrium with good news, and we know that in this case $p_*^-$ Pareto dominates any other equilibrium (Lemma 9). When $c \leq \underline{c}$, equilibriums with good news disclosure and bad news disclosure may coexist. However, from Lemma 16,when this is the case, the equilibrium with disclosure of bad news is Pareto dominant. ∎

## C.3 General Case

# Proof of Proposition 6

*Proof.* First, we verify that the disclosure strategy is optimal whenever $V_t = 0$. By construction $U_0(p_*) = U_0(0)$ so the manager is indifferent between disclosing negative information or not when $p_t = p_*$. Moreover, given that $U_0(p)$ is non-decreasing, the manager does not have incentives to deviate and disclose if $p_t > p_*$.

Next, we verify that the disclosure strategy is also optimal when $V_t = 1$. The manager disclosure strategy is optimal if the following two conditions are satisfied

(1) $U_1(1) \geq c$.

(2) $U_1(p) \geq U_1(1) - c$ for $p \geq p_*$.

For (1) note that, by construction (equation (30)),

$$U_1(1) - c = \left(1 + \frac{r}{\lambda_1}\right) U_0(p_*) = \left(1 + \frac{r}{\lambda_1}\right)\left(\frac{p_*}{r} - \frac{\mu\theta}{r}\frac{r+\lambda_0}{r+\kappa}\right),$$

which is always positive given the assumption that

$$p_* \geq \mu\theta \frac{r+\lambda_0}{r+\kappa}.$$

For (2), note that as $U_1$ is increasing (2) is satisfied if and only if $U_1(p_*) \geq U_1(1) - c$. This happens if and only if

$$\left(1 + \frac{r}{\lambda_1}\right)\left(\frac{p_*}{r} - \frac{\mu\theta}{r}\frac{r+\lambda_0}{r+\kappa}\right) \leq \frac{p_*}{r} - \frac{\mu\theta}{r}\frac{\lambda_0}{r+\kappa},$$

which is true for all $p_* \leq \mu\theta$.

The only step left is to show that $U_0, U_1$ are nondecreasing functions of $p$. By construction, $U_1$ is not differentiable at $p_*$ (is not even continuous). Let $U'_{1+}(p_*)$ and $U''_{1+}(p_*)$ be the right hand first and second derivative, respectively. Similarly, let $U''_{0+}(p_*)$ be the right hand second derivative of $U_0$ at $p_*$. Evaluating the HJB equation at $p_*$ and using the initial conditions (28) and (29) we get that $U'_0(p_*) = 0$ and

$$U'_0(p) = 0 \Rightarrow U''_{0+}(p) = \frac{1 + \lambda_1 U'_{1+}(p)}{-f(p)}. \tag{71}$$

Using the HJB equation for $U_1$ and the initial conditions we get (28) and (29) we get

$$f(p_*)U'_{1+}(p_*) = 0.$$

By assumption $p_* > \hat{p}$ so $f(p_*) < 0$ which means that $U'_{1+}(p_*) = 0$. Differentiating the HJB equation for $U_1$ we get

$$U'_{1+}(p) = 0 \Rightarrow U''_{1+}(p) = \frac{1 + \lambda_0 U'_{0+}(p)}{-f(p)}. \tag{72}$$

In particular, we get that $U'_{1+}(p_*) = 0$. Using (71) and (72) we find that $U''_{0+}(p_*) > 0$ and $U''_{1+}(p_*) > 0$. Which means that there $\epsilon > 0$ such that $U'_0(p) > 0$ and $U'_1(p) > 0$ for $p \in (p_*, p_* + \epsilon)$.[30] Let $\tilde{p} := \inf\{p > p_* | U'_0(p) < 0 \text{ or } U'_1(p) < 0\}$. As $U_0$ and $U_1$ are continuously differentiable in $(p_*, 1)$, we have that either $U'_0(\tilde{p}) = 0$ and $U'_1(\tilde{p}) \geq 0$

[30] $U_1$ is twice continuously differentiable for $p > p_*$.

or $U_1'(\tilde{p}) = 0$ and $U_0'(\tilde{p}) \geq 0$. Without loss of generality, suppose that $U_0'(\tilde{p}) = 0$ and $U_1'(\tilde{p}) \geq 0$. Equation (71) implies that $U_0''(\tilde{p}) > 0$. Suppose that $U_1'(p) > 0$, then there is $\tilde{\epsilon}$ such that $U_0'(p) \geq 0$ and $U_1'(p) \geq 0$ for all $p \in (\tilde{p}, \tilde{p} + \tilde{\epsilon})$ contradicting the definition of $\tilde{p}$ as the $\inf\{p > p_* | U_0'(p) < 0 \text{ or } U_1'(p) < 0\}$. On the other hand, if $U_1'(\tilde{p}) = 0$ then equation (72) implies that $U_1''(\tilde{p}) > 0$ which means that we can also find $\tilde{\epsilon}$ such that $U_0'(p) \geq 0$ and $U_1'(p) \geq 0$ for all $p \in (\tilde{p}, \tilde{p} + \tilde{\epsilon})$. Hence, $U_0$ must be nondecreasing in $[p_*, 1]$. A symmetric argument can be used to show that $U_1$ is nondecreasing. ∎